

# Electronique quantique dans les nanoconducteurs

## Experiences LPENS

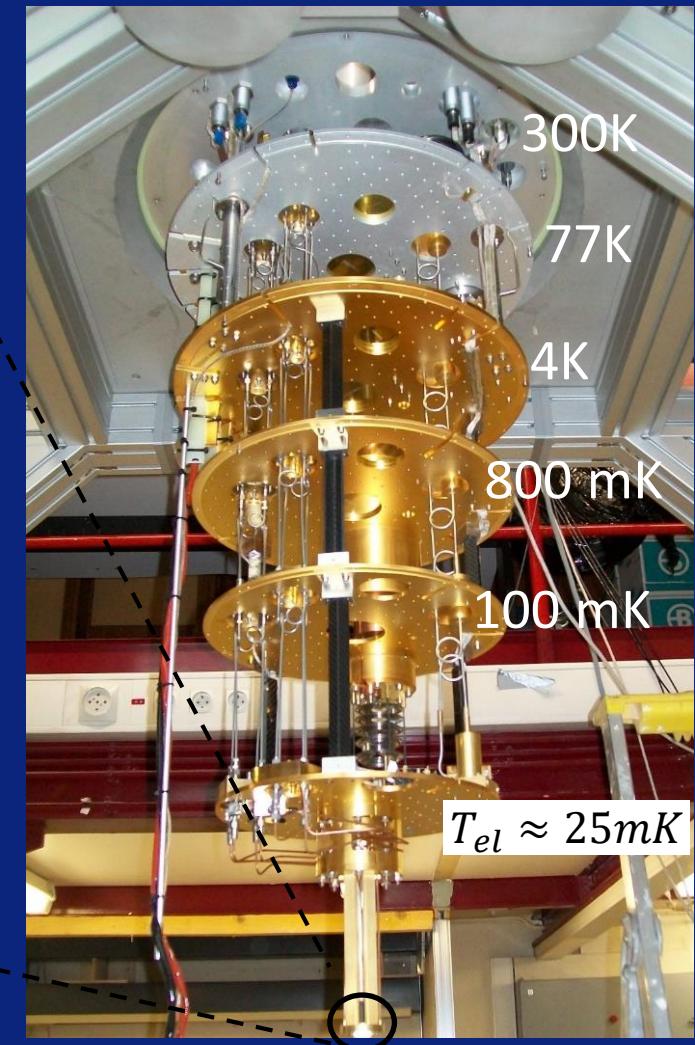
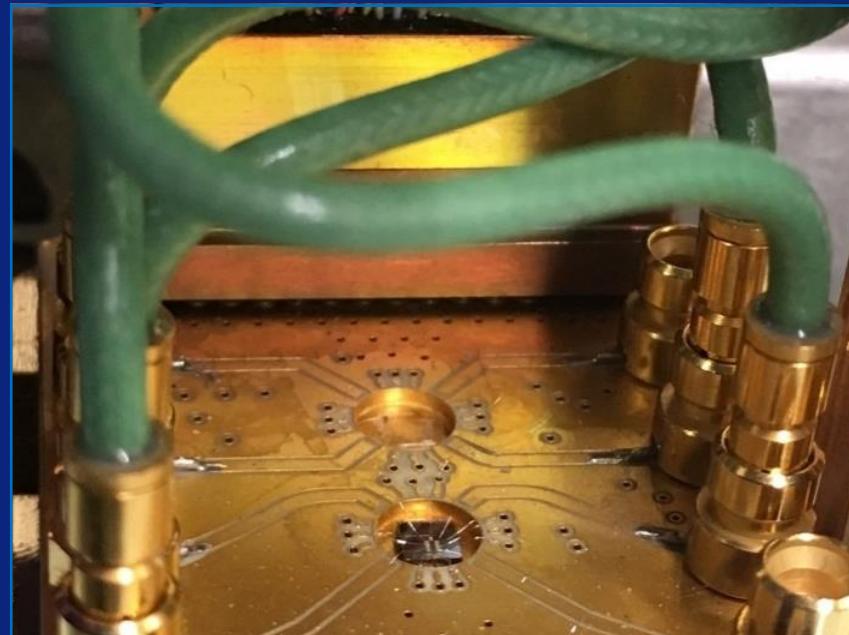
H. Bartolomei, M. Kumar,  
A. Marguerite, R. Bisognin,  
J.M Berroir, E. Bocquillon,  
B. Plaçais, G. Fèvre

## Echantillons, C2N Palaiseau

Y. Jin, Q. Dong, A. Cavanna,  
U. Gennser

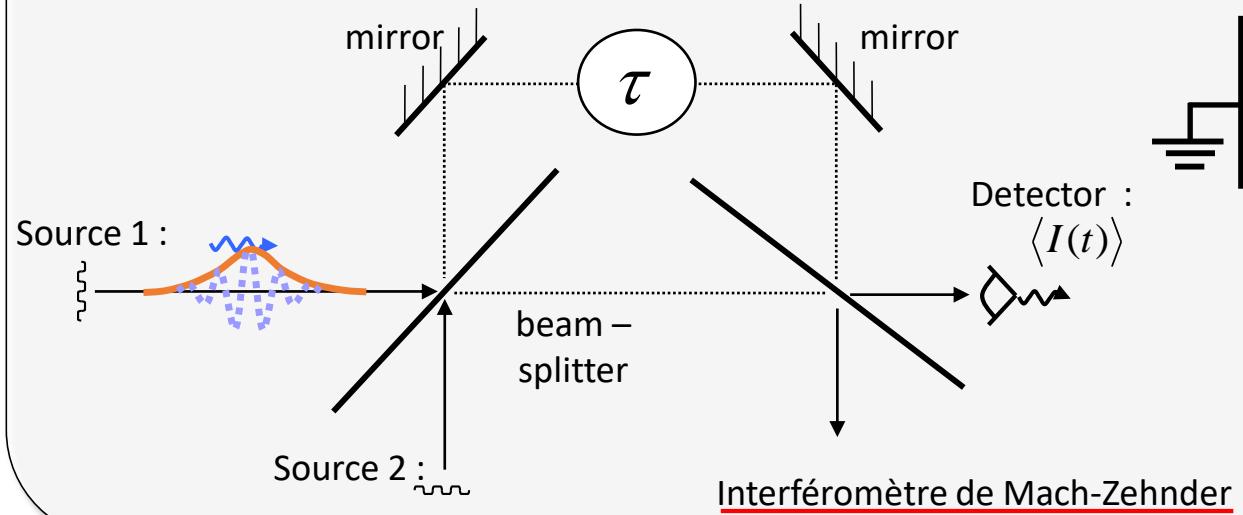
## Collaborations théoriques:

CPT Marseille, LPENS Lyon,  
LPS Orsay

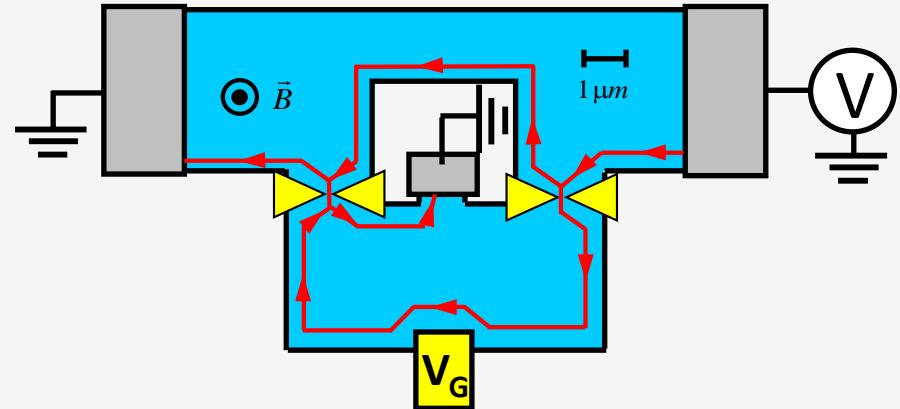


# Optique électronique dans les conducteurs de Hall quantique

- Interférométrie à 1 particule

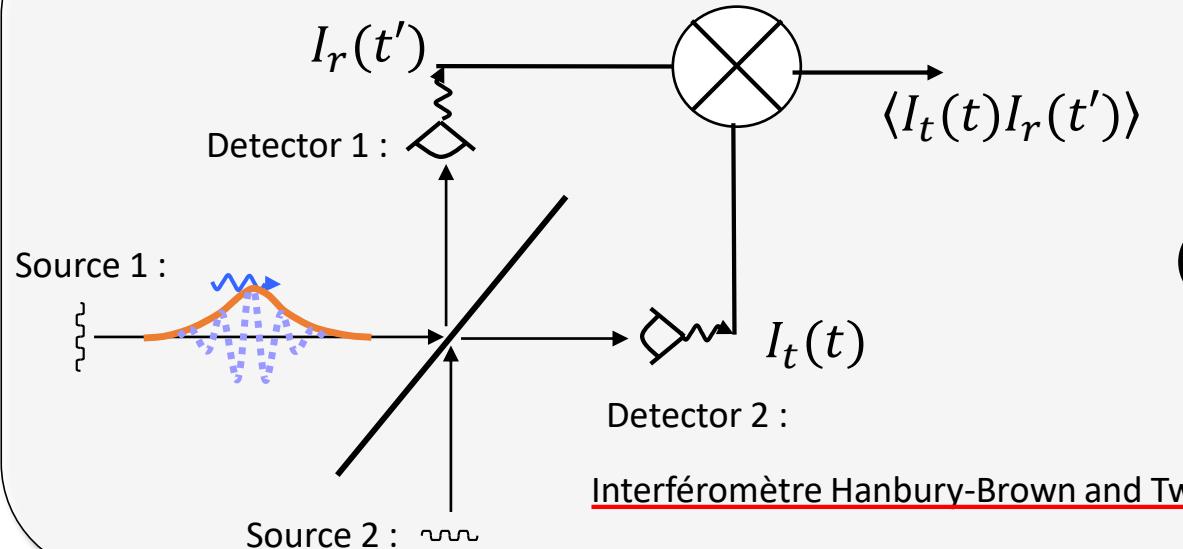


Interféromètre de Mach-Zehnder

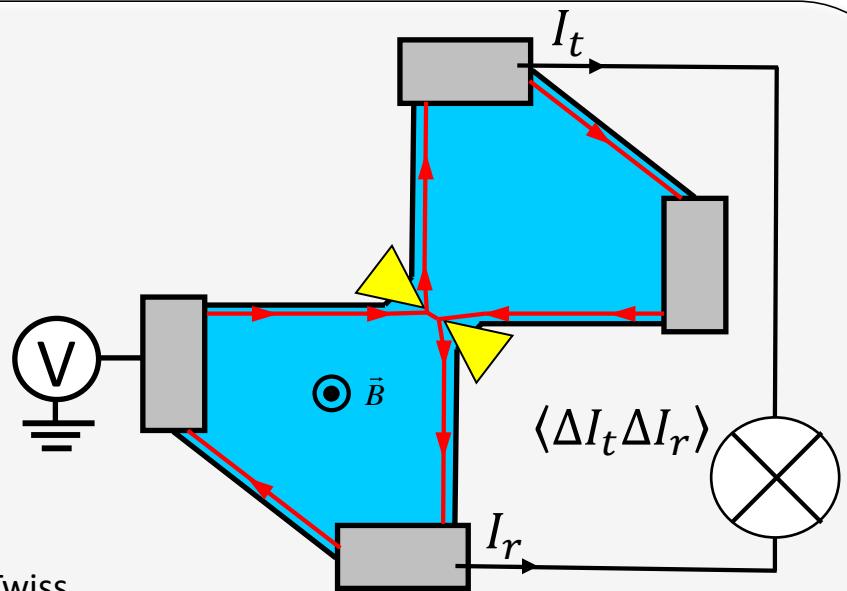


Y. Ji et al., Nature **422**, 415 (2003)

- Interférométrie à 2 particules



Interféromètre Hanbury-Brown and Twiss

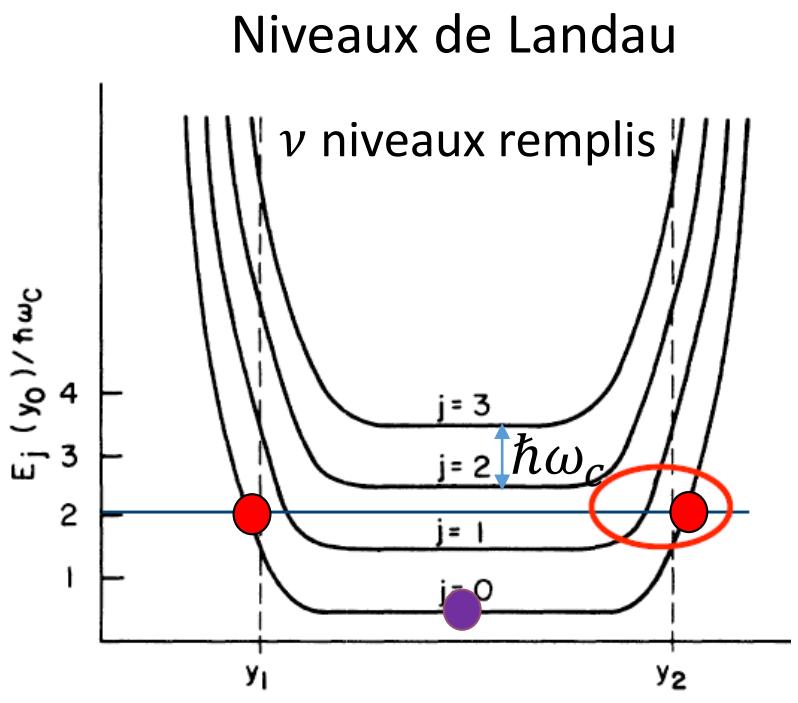
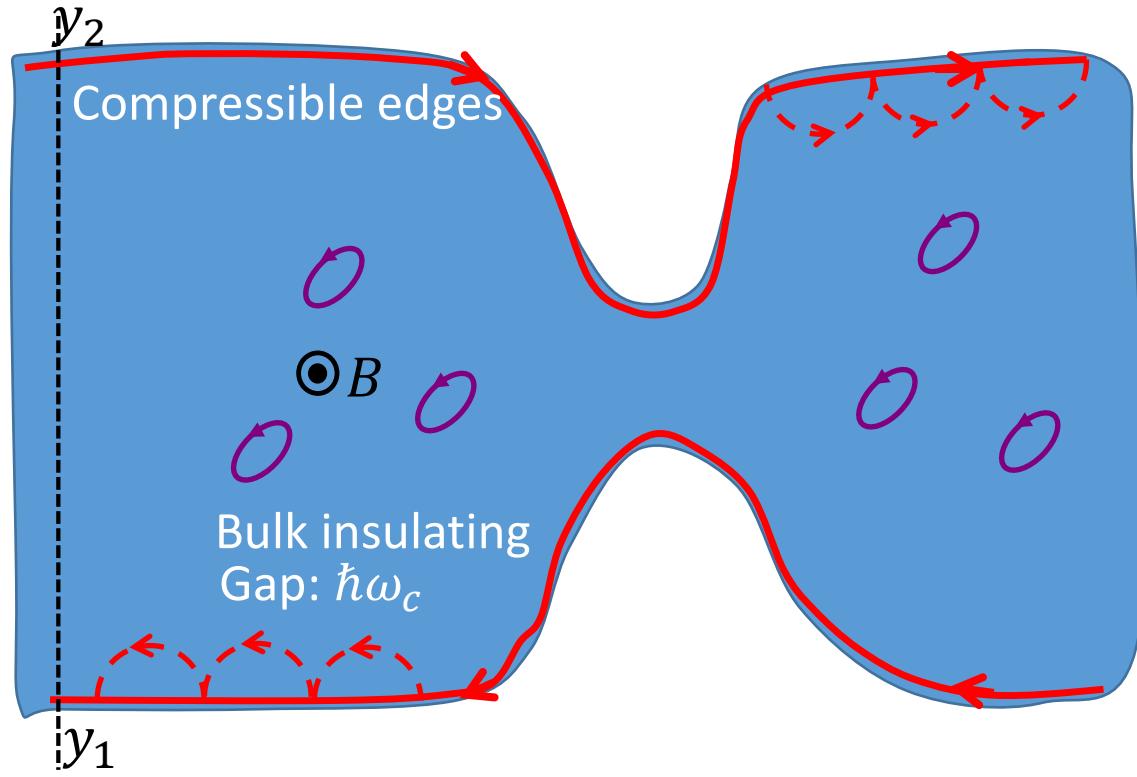
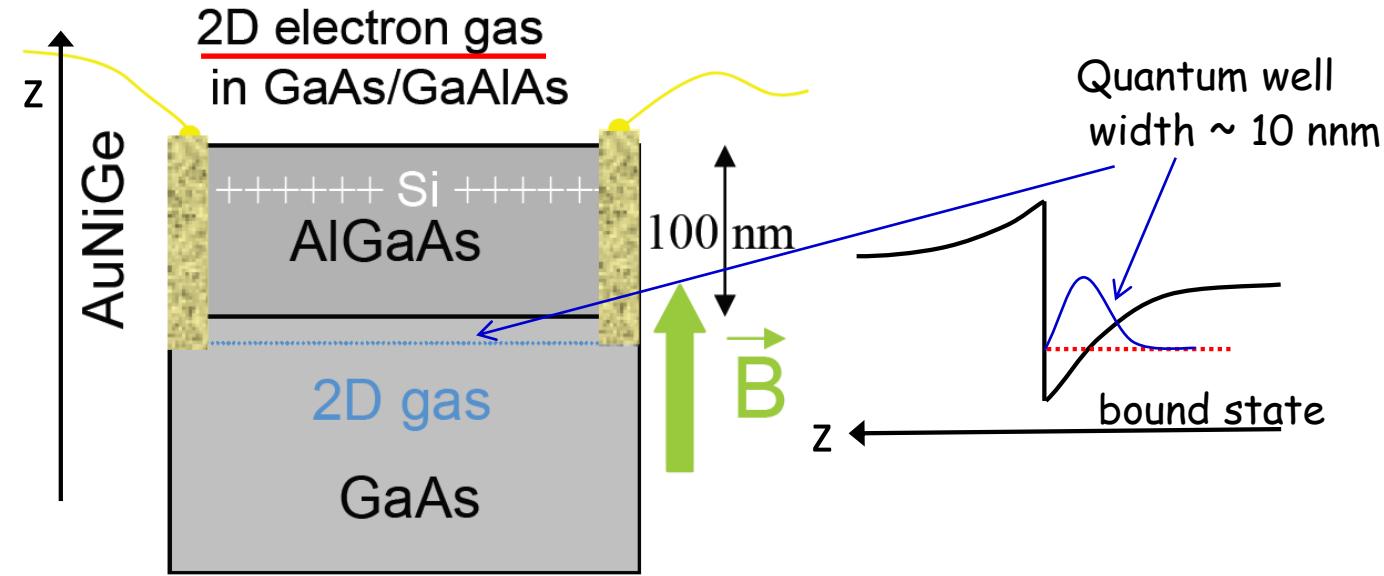


M. Henny et al., Science **284**, 296 (1999)

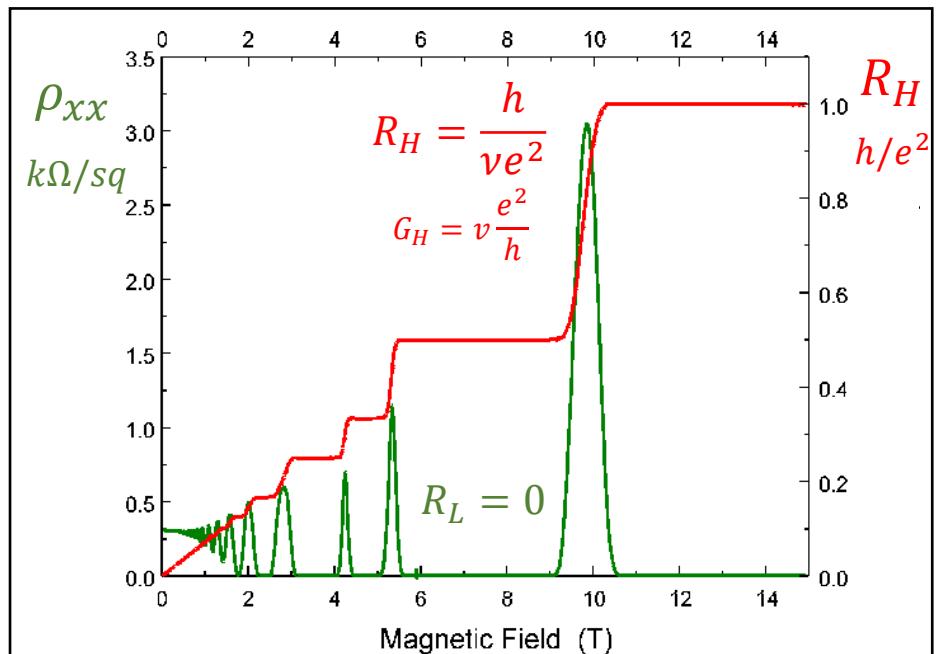
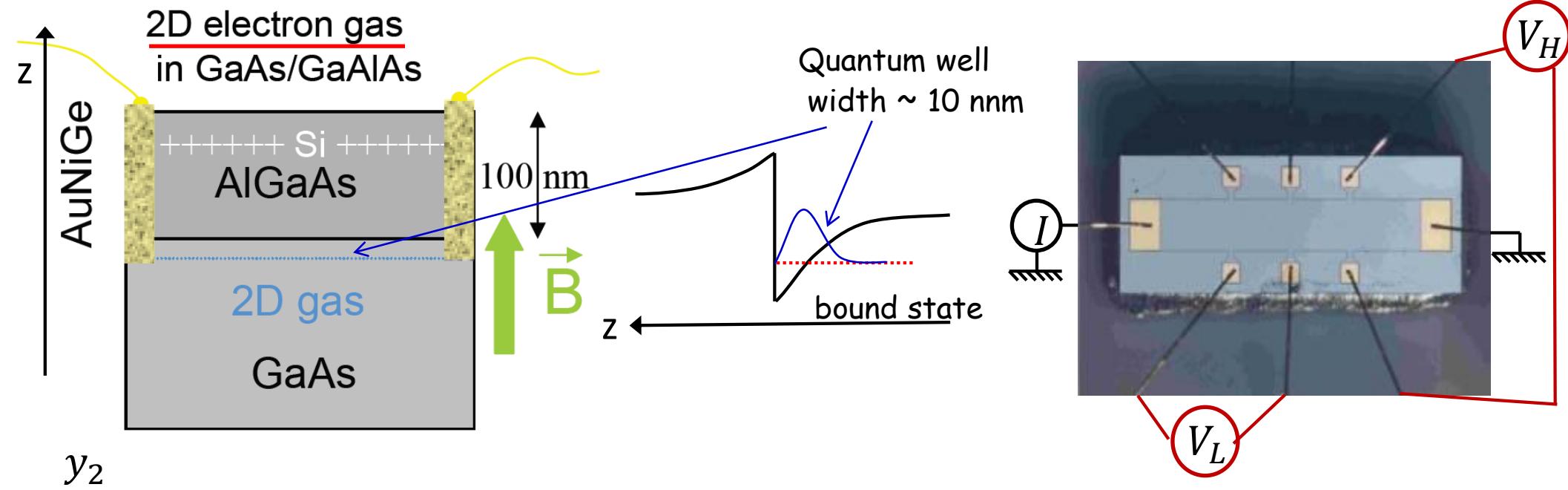
# I Electrons dans le régime d'effet Hall quantique entier

# II Anyons dans le régime d'effet Hall quantique fractionnaire

# Matériaux 2D et fort champ magnétique: l'effet Hall quantique

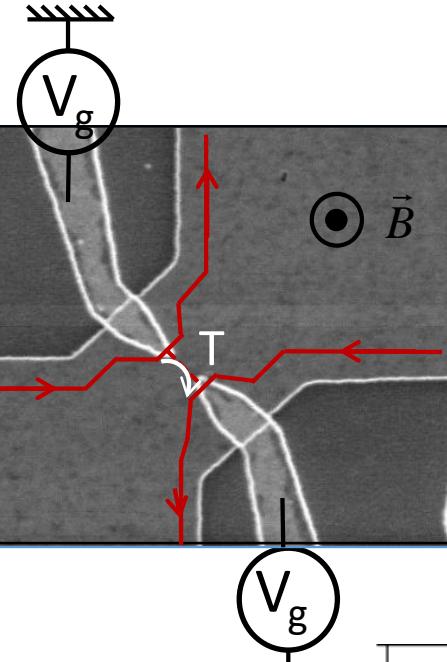
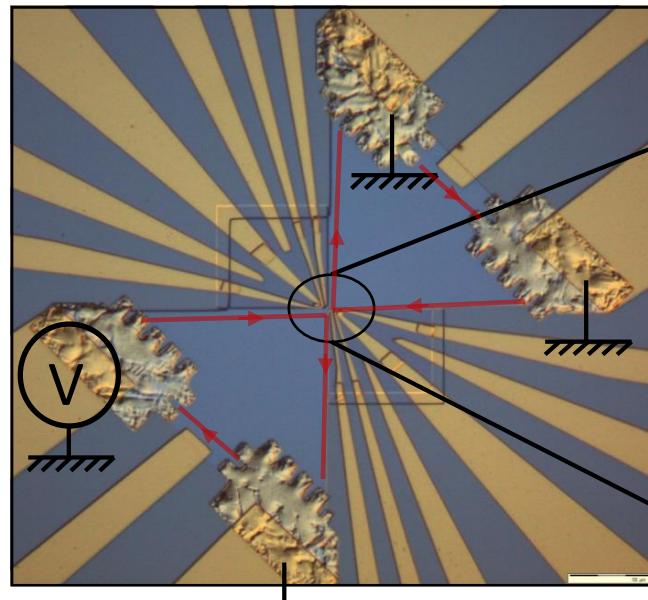


# Matériaux 2D et fort champ magnétique: l'effet Hall quantique

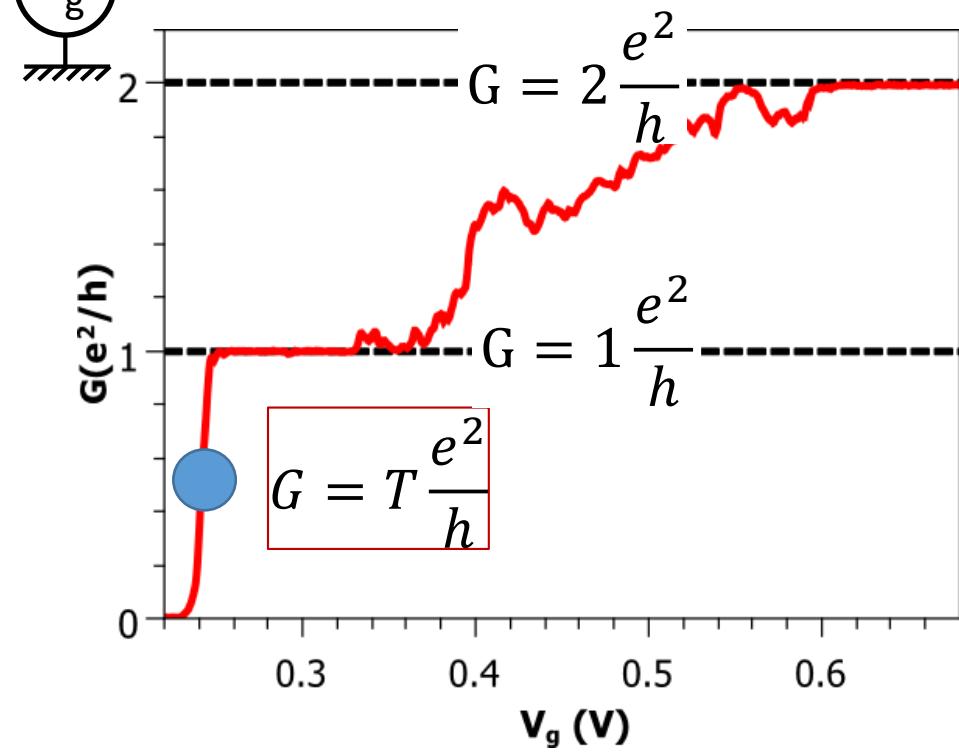
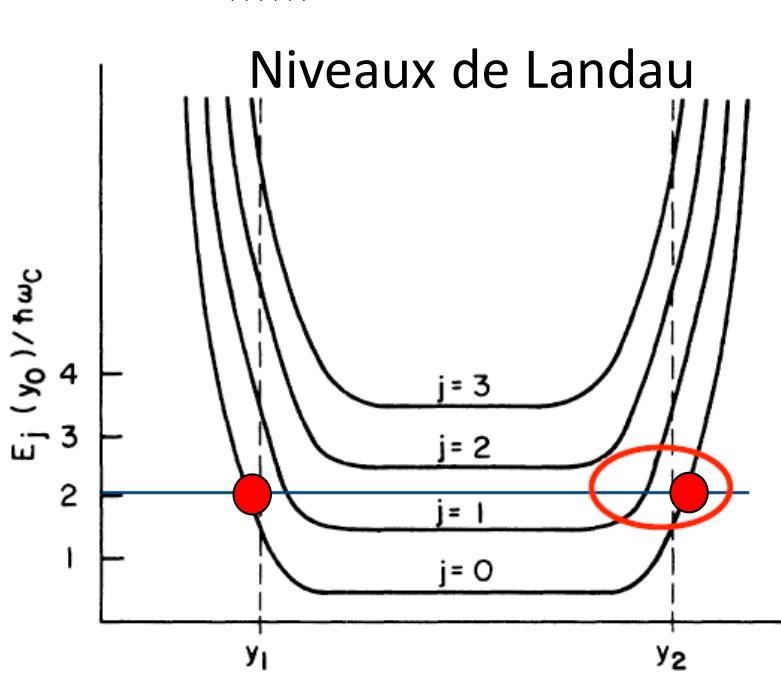


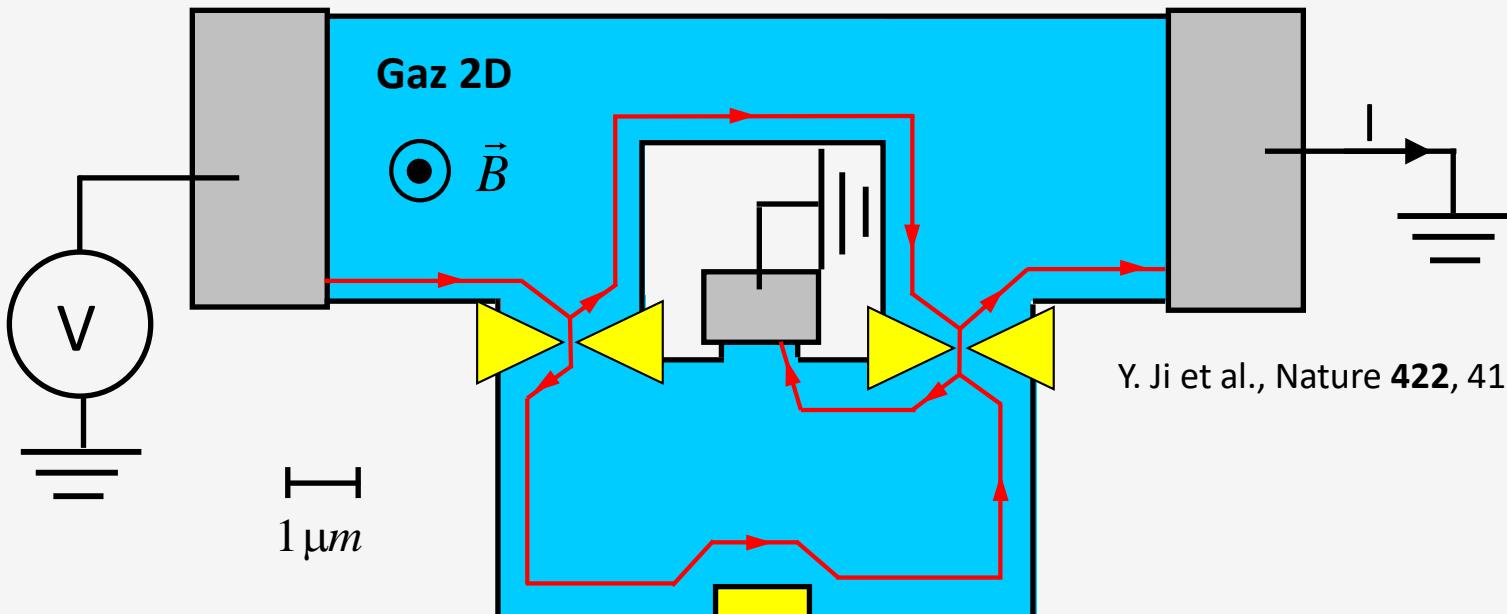
**Effet Hall quantique:**  
K. v. Klitzing, G. Dorda, and M. Pepper,  
Phys. Rev. Lett. **45**, 494 (1980).  
  
Quantification robuste de la résistance Hall  
 $v$  fils 1D transportant le courant sans  
rétrodiffusion

# Le contact ponctuel quantique: une lame semi-réfléchissante réglable

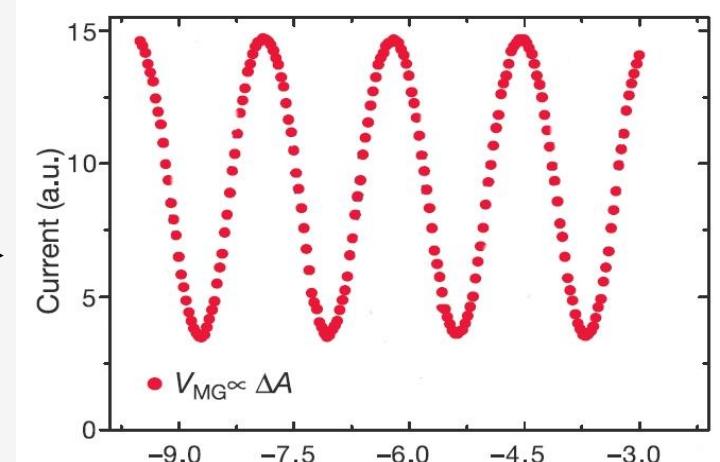
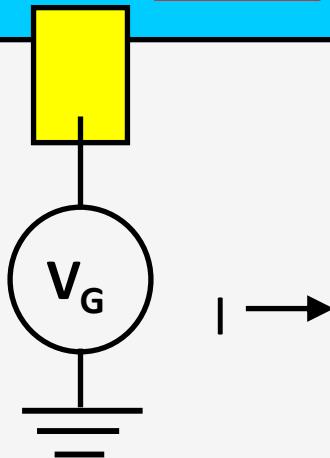
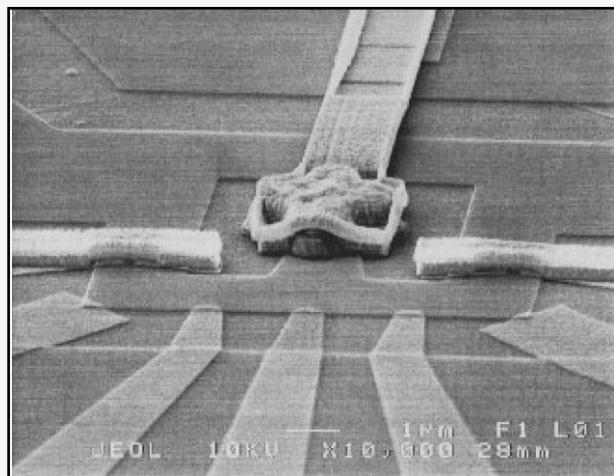


Example,  $\nu = 2$



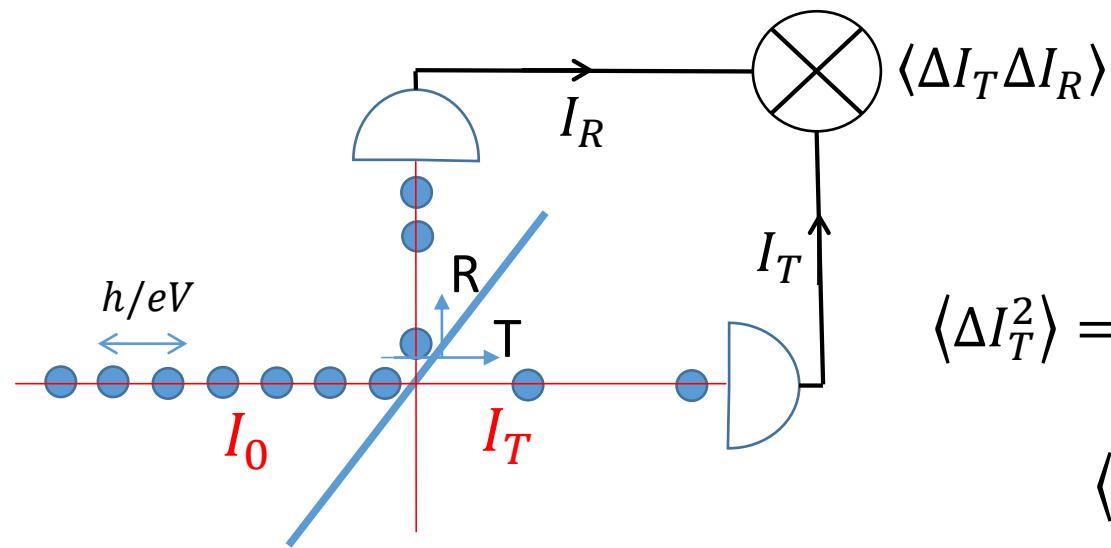


Y. Ji et al., Nature **422**, 415 (2003)



$V_G$  (mV)

# Interféromètre à deux particules: mesures de bruit

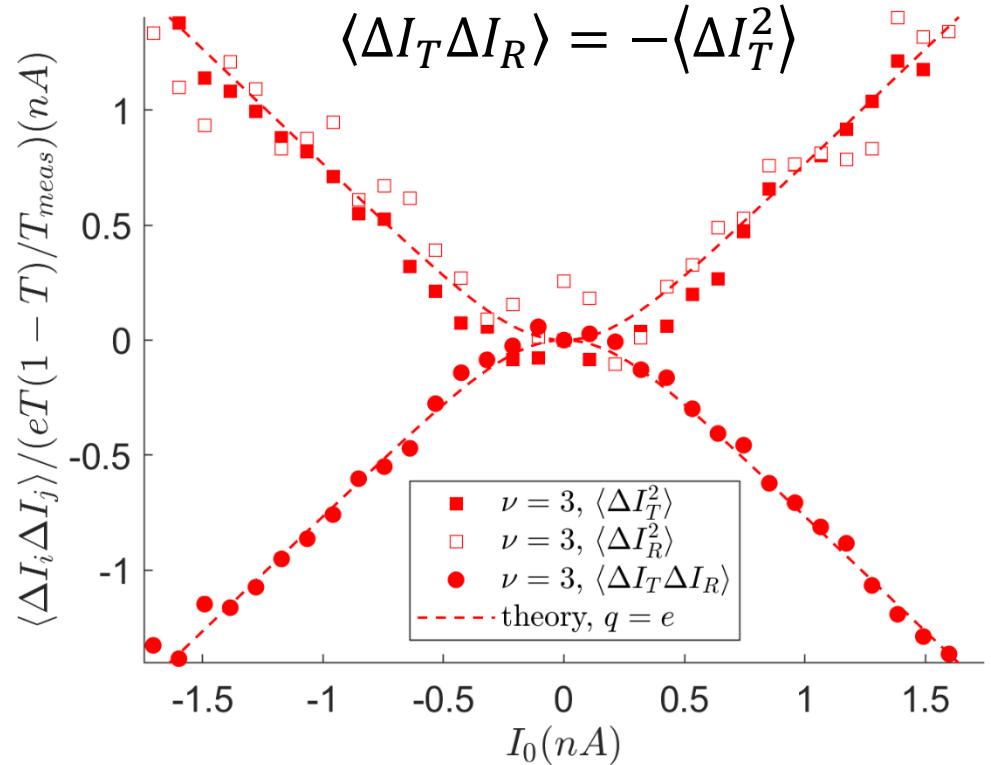
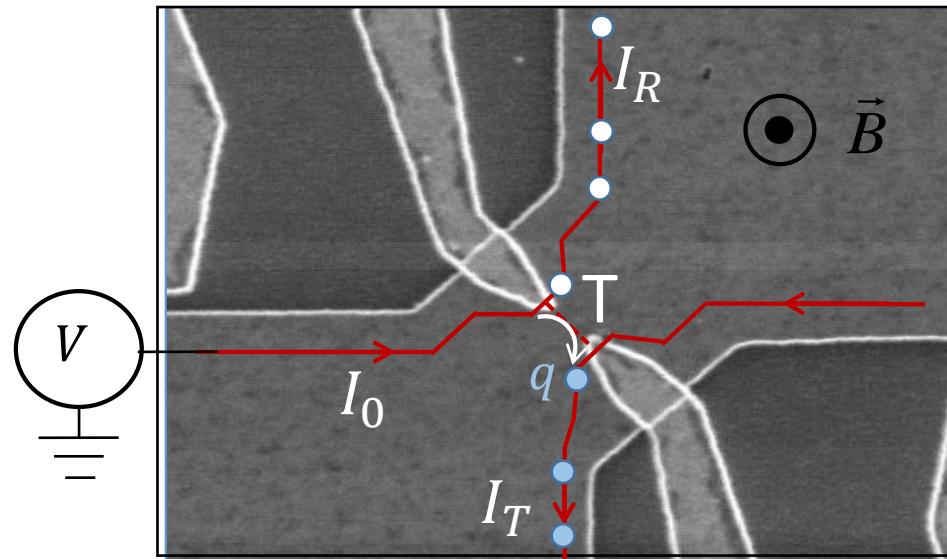


$T \ll 1$ : processus de Poisson

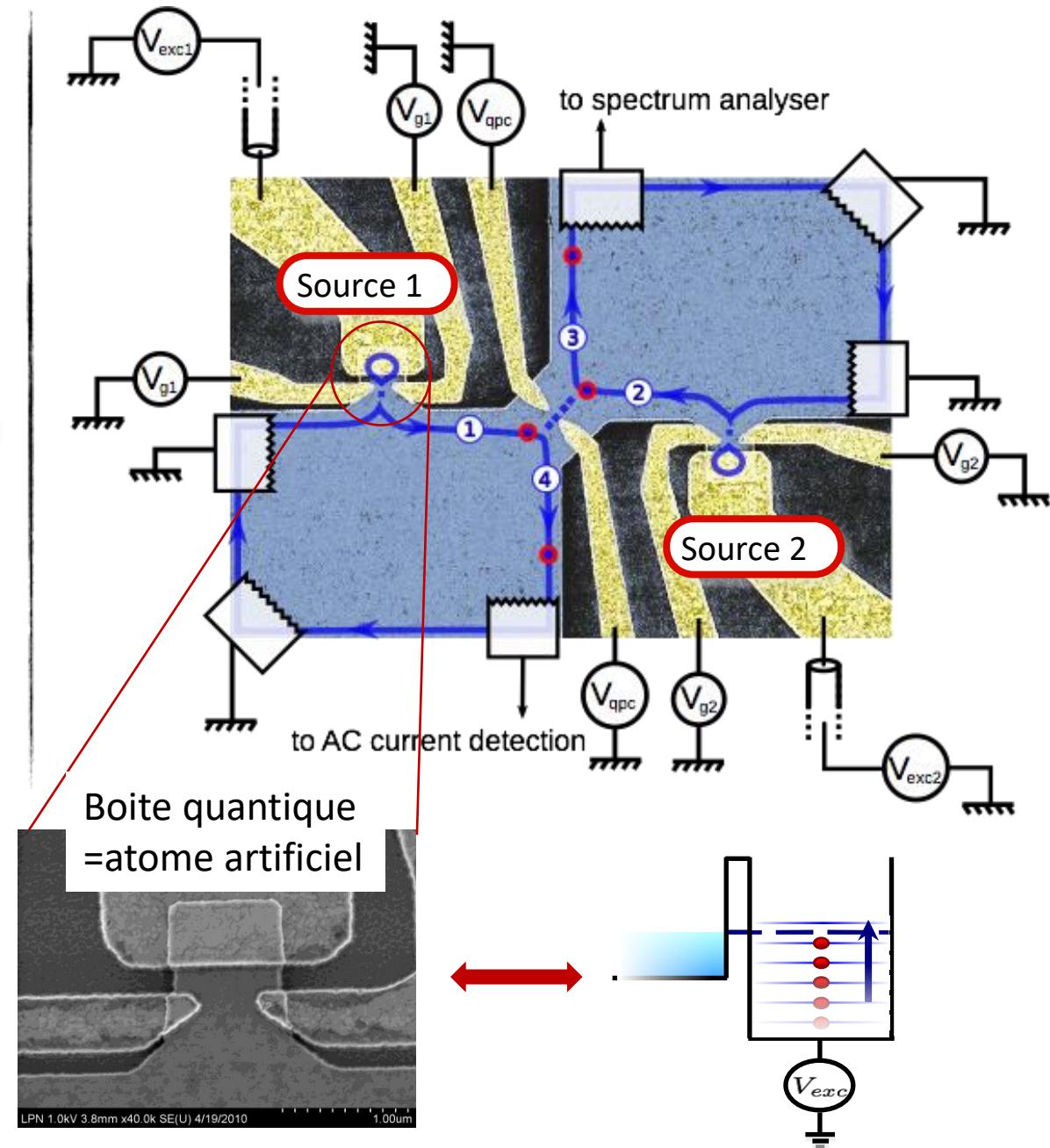
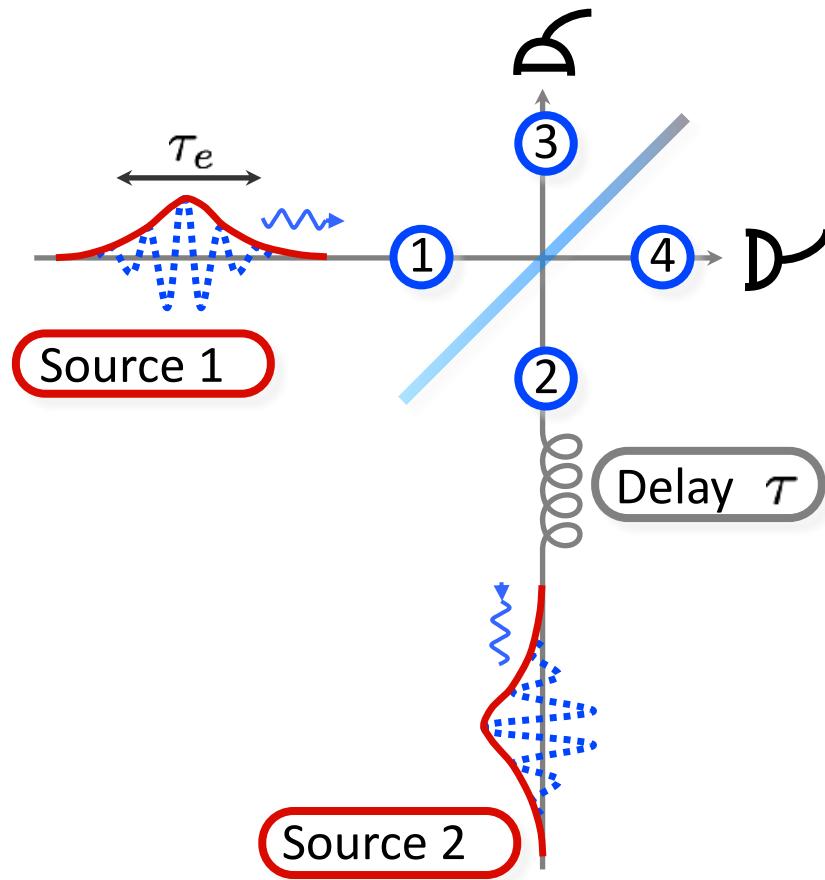
$$\langle \Delta N_T^2 \rangle = \langle N_T \rangle = TN_0$$

$$\langle \Delta I_T^2 \rangle = \frac{e^2}{T_{meas}^2} \langle \Delta N_T^2 \rangle = \frac{eT}{T_{meas}} \frac{eN_0}{T_{meas}} = \frac{eT}{T_{meas}} I_0$$

$$\langle \Delta I_T^2 \rangle + \langle \Delta I_R^2 \rangle + 2\langle \Delta I_T \Delta I_R \rangle = \langle \Delta I_0^2 \rangle = 0$$



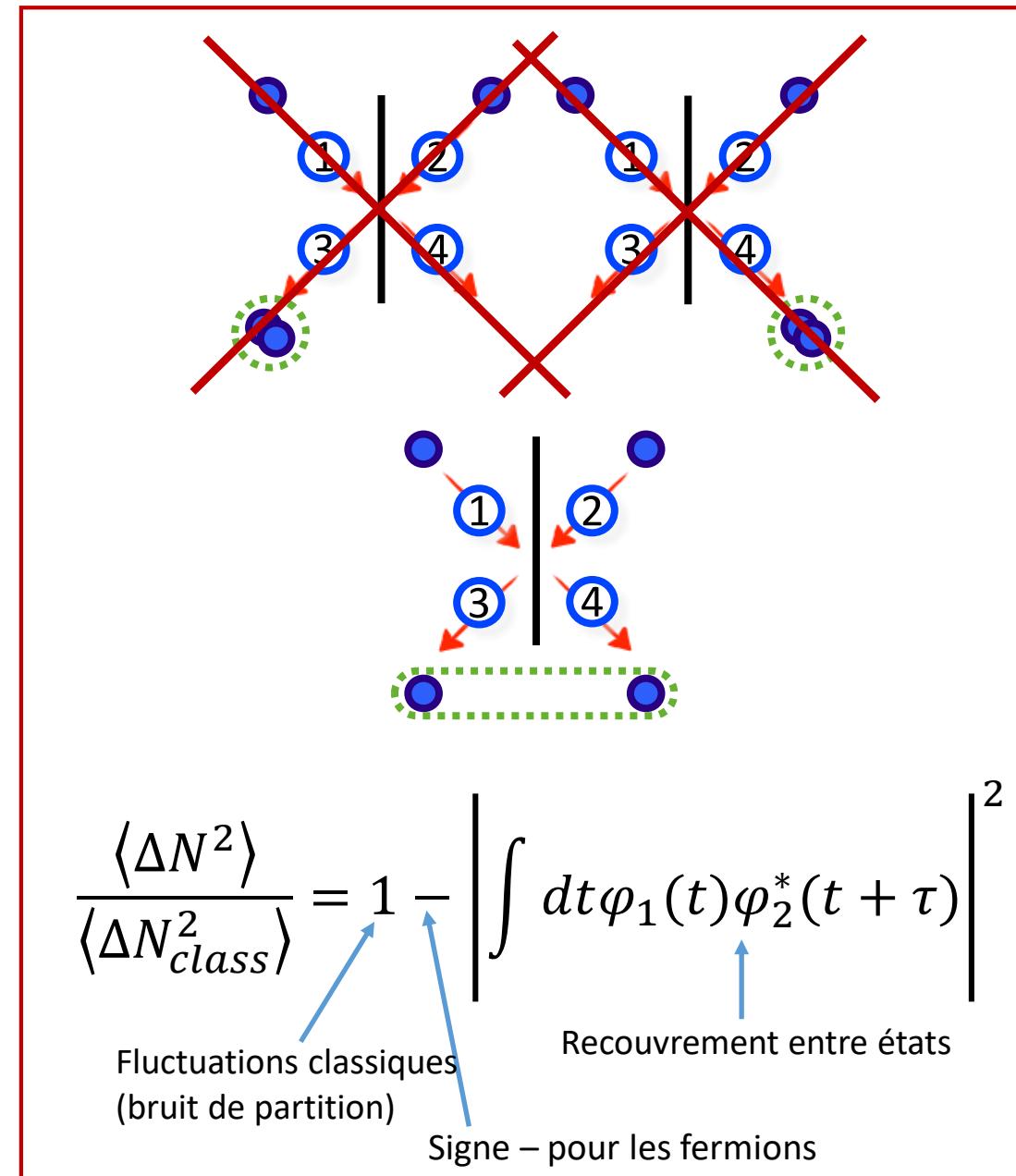
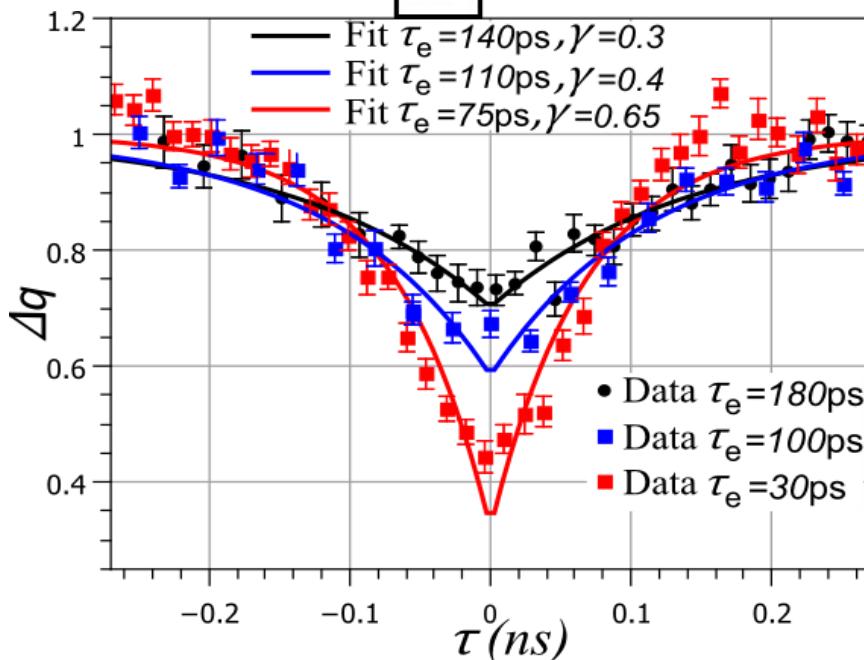
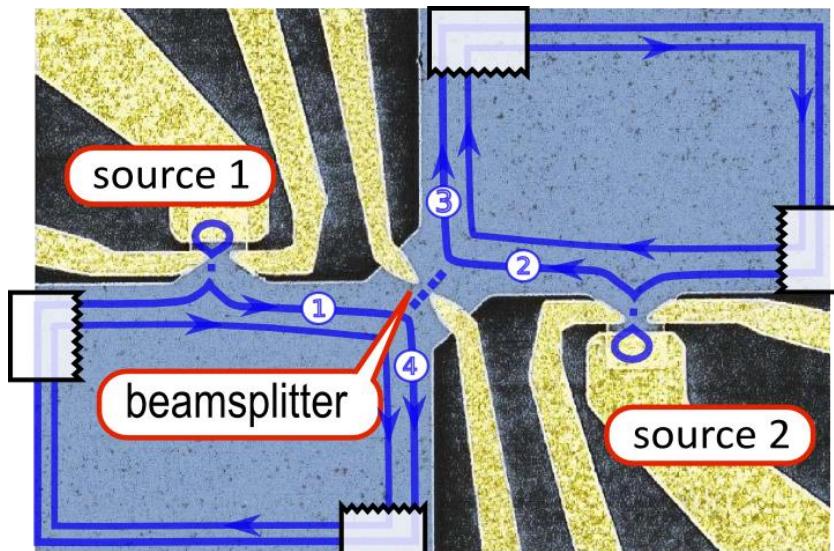
# Interféromètre à deux particules: l'expérience HOM électronique



**Photons :**

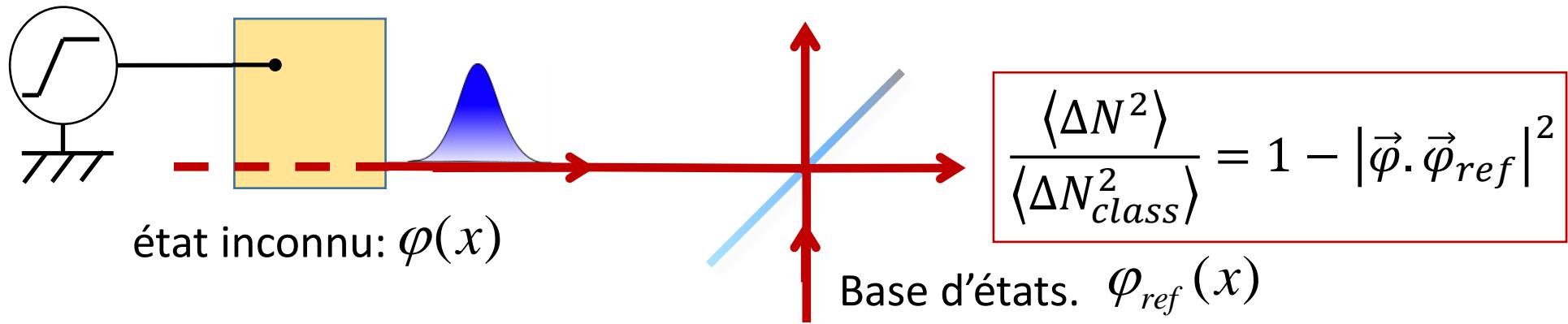
C. Hong *et al.*, PRL 59(18), 2044 (1987)

# Noise measurements and two electron interferences



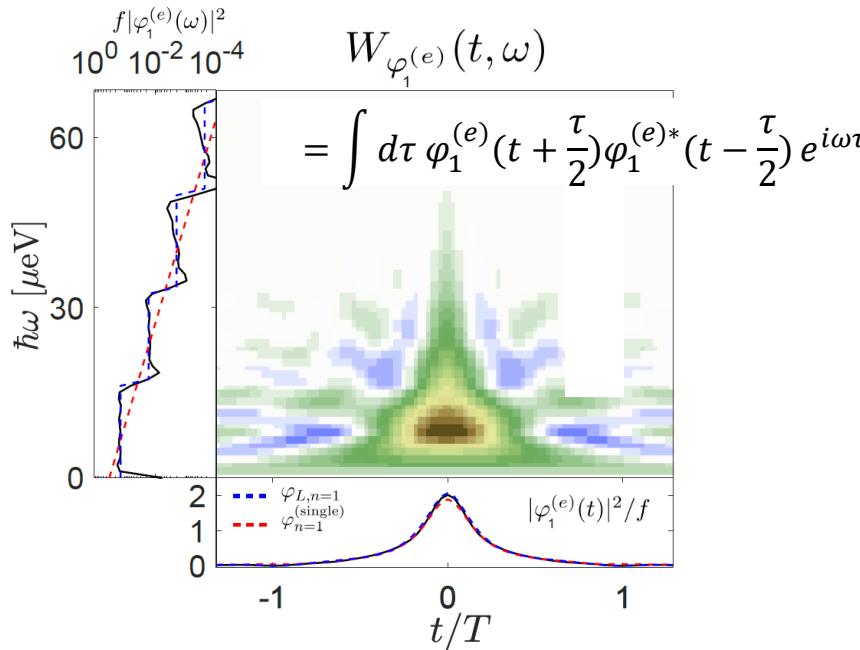
- S. Ol'khovskaya et al., PRL **101**, 166802, (2008).  
 E. Bocquillon et al., Science **339**, 1054 (2013).  
 A. Marguerite et al., PRB **94**, 115311 (2016).

# Extraction des fonctions d'onde d'électrons

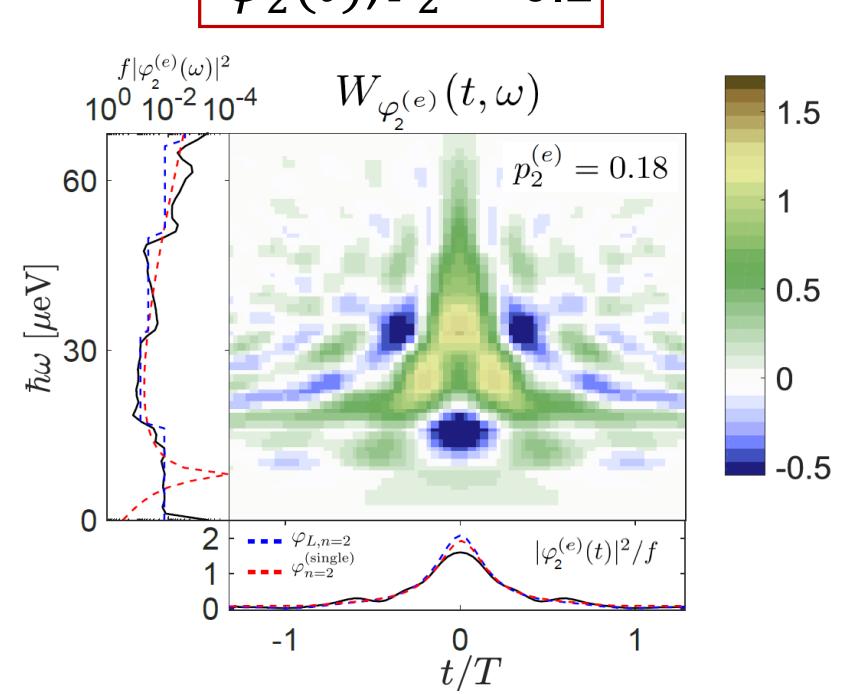


Mélangé:

$$\varphi_1(t), P_1 = 0.8$$



$$\varphi_2(t), P_2 = 0.2$$

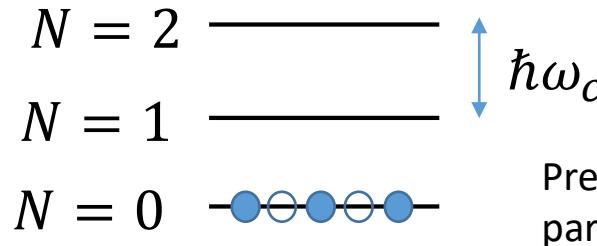


I Electrons dans le régime d'effet Hall  
quantique entier

II Anyons dans le régime d'effet Hall  
quantique fractionnaire

# L'effet Hall quantique fractionnaire

High field,  $\nu < 1$



La seule échelle d'énergie restante est l'interaction de Coulomb  $H = E_c$

On retrouve un système isolant au cœur (incompressible) pour des remplissages spécifiques:

$$\nu = \frac{1}{m} \left( \frac{1}{3}, \dots \right)$$

R. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

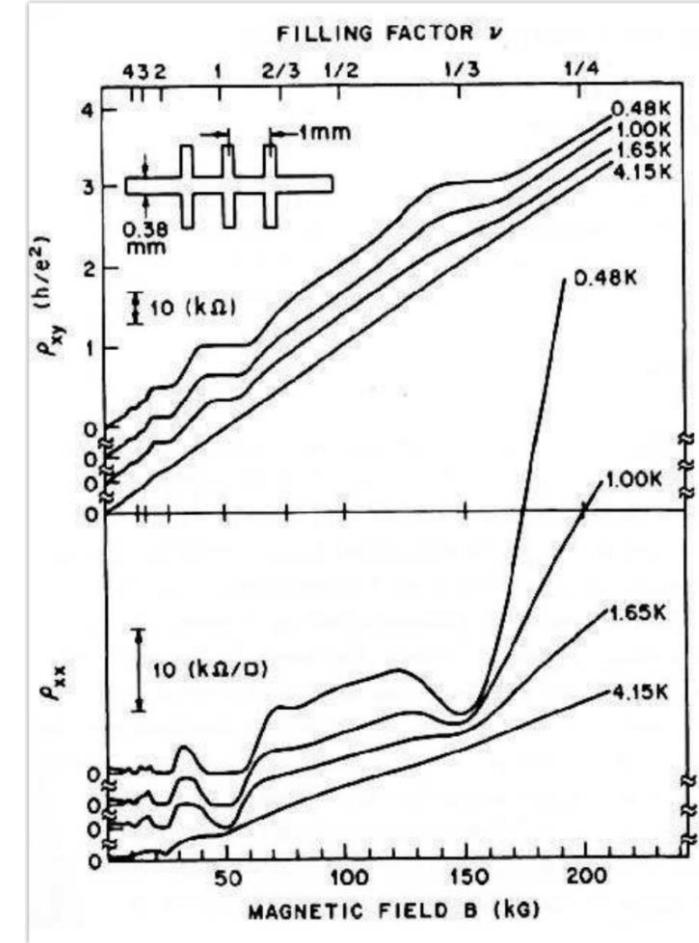
Résistance/ conductance Hall quantifiée

$$G_H = \frac{1}{m} \frac{e^2}{h}$$

Excitations élémentaires= anyons: charge et statistique fractionnaires

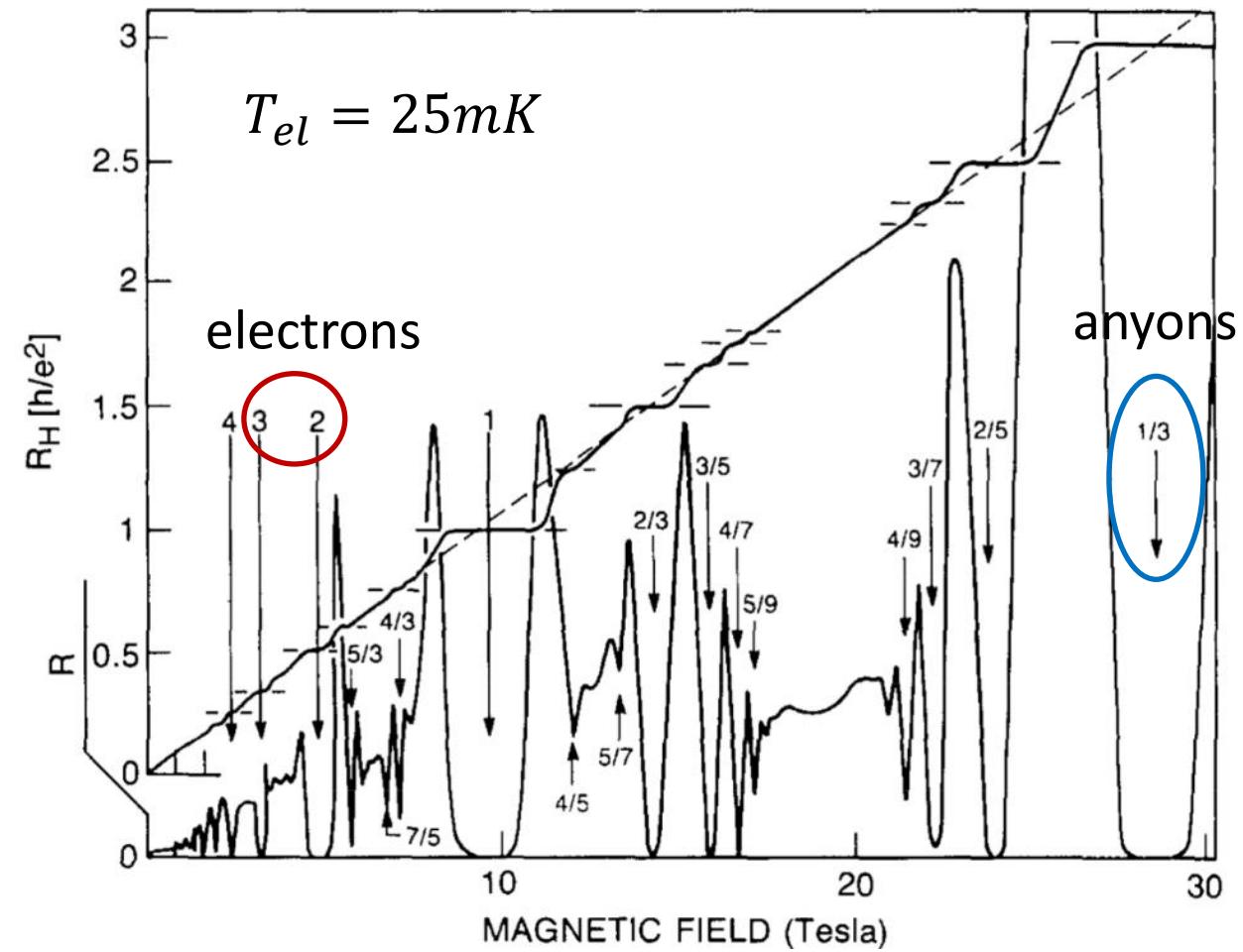
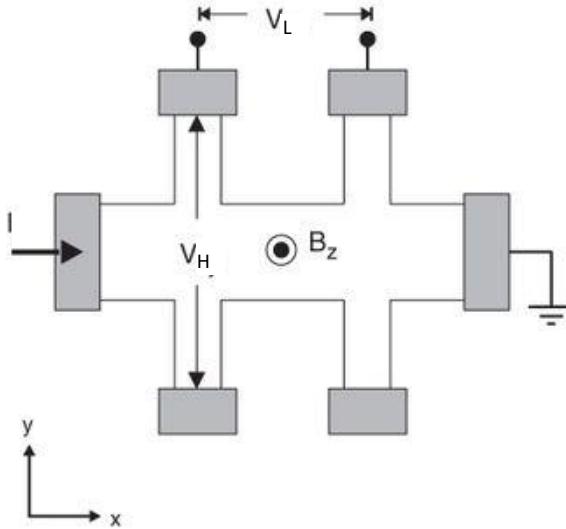
$$\nu = \frac{1}{m}, \quad e^* = \frac{e}{m}, \quad \varphi = \frac{\pi}{m}$$

Halperin, PRL **52** 1583 (1984)



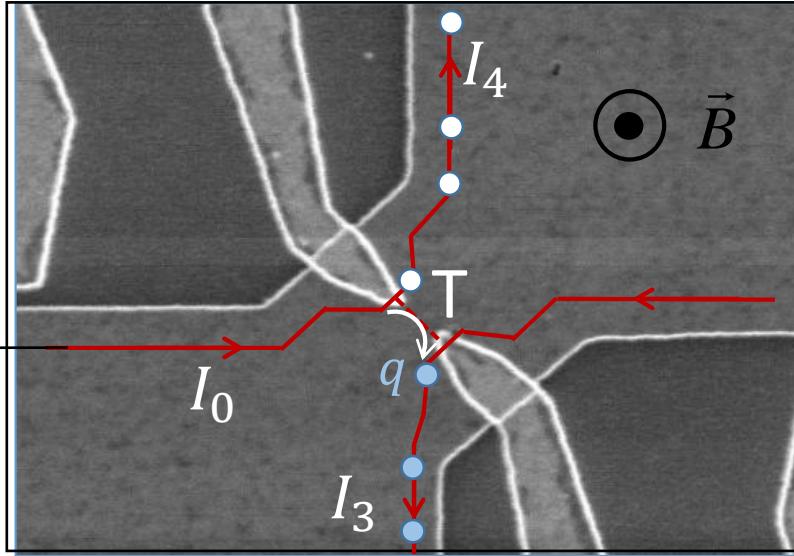
D.C. Tsui, H.L. Stormer, and A.C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

# L'effet Hall quantique fractionnaire



H.L. Stormer, Physica B **177**, 401 (1992).

# Bruit et charges fractionnaires, $\nu=1/3$

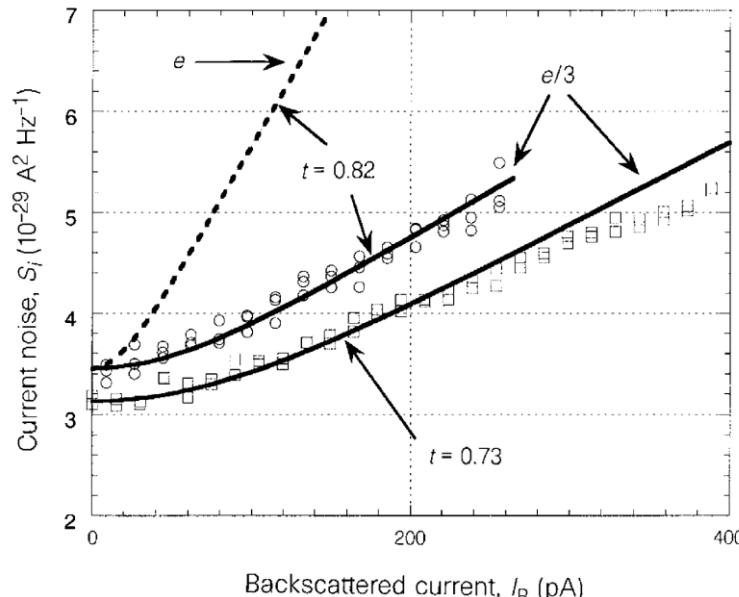


$T \ll 1$ : processus de Poisson

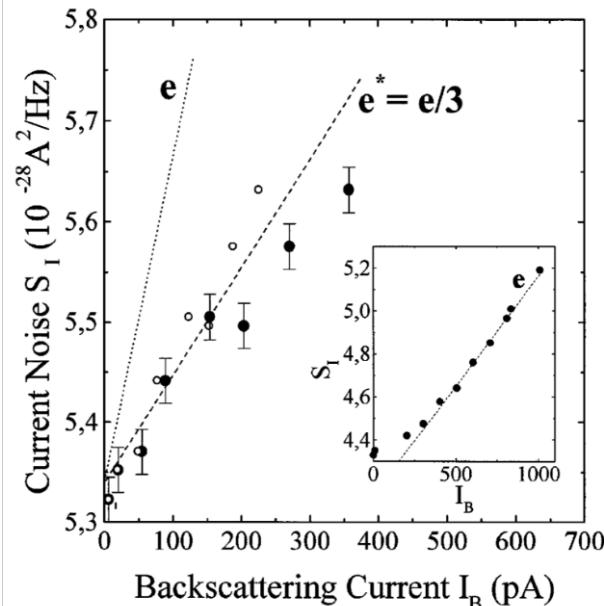
$$\langle \Delta N_T^2 \rangle = \langle N_T \rangle = TN_0$$

$$\langle \Delta I_T^2 \rangle = \frac{q^2}{T_{meas}^2} \langle \Delta N_T^2 \rangle = \frac{qT}{T_{meas}} \frac{qN_0}{T_{meas}} = \frac{qT}{T_{meas}} I_0$$

Fractional case:  $\nu = 1/3, q = e/3$

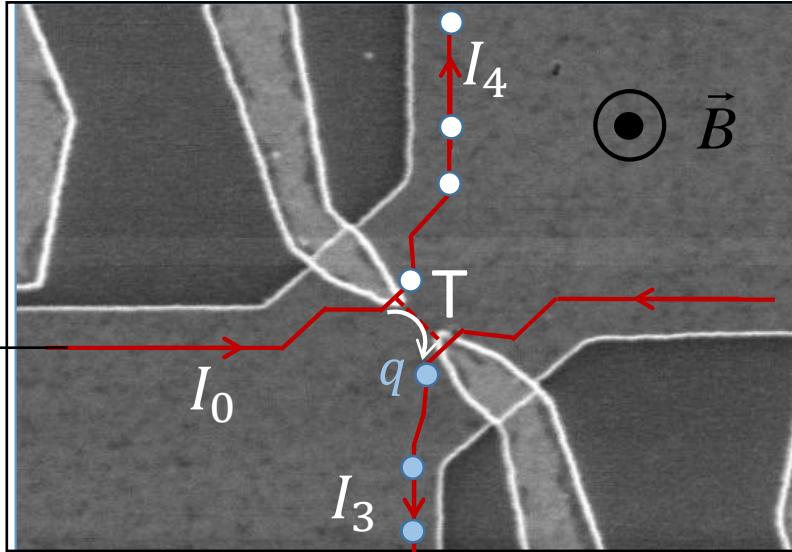


R. de Picciotto et al., Nature 389, 162 (1997).



L. Saminadayar et al., Phys. Rev. Lett. 79, 2526 (1997).

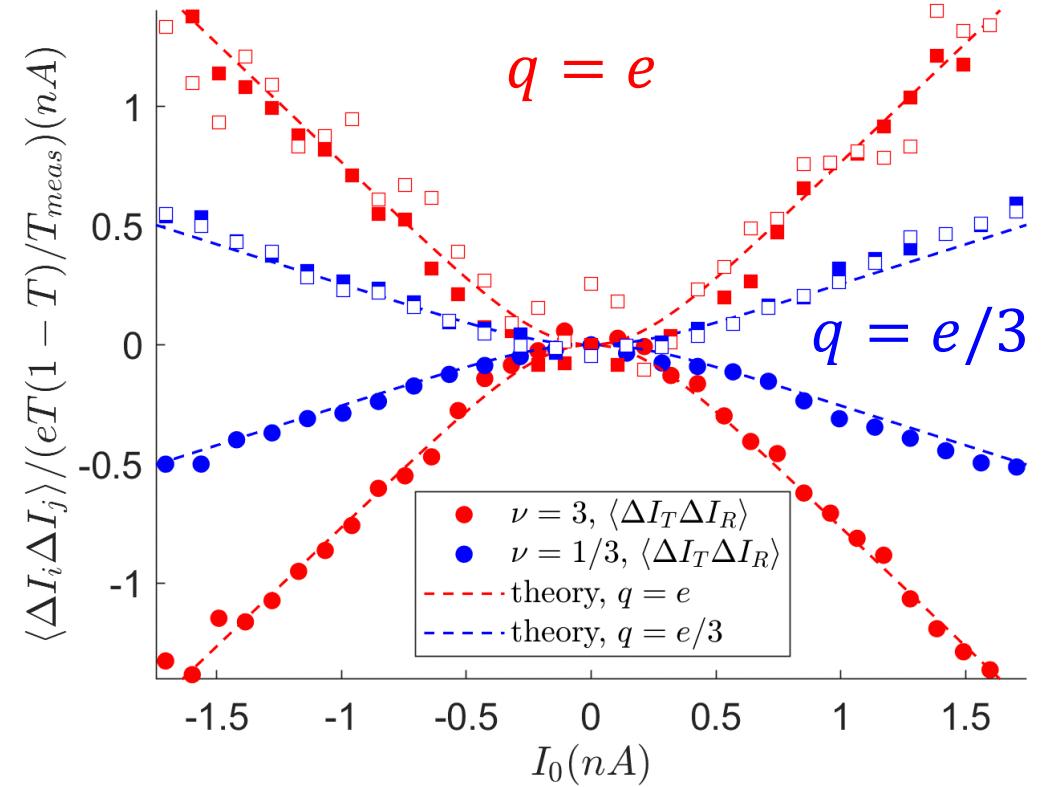
# Bruit et charges fractionnaires, $\nu=1/3$

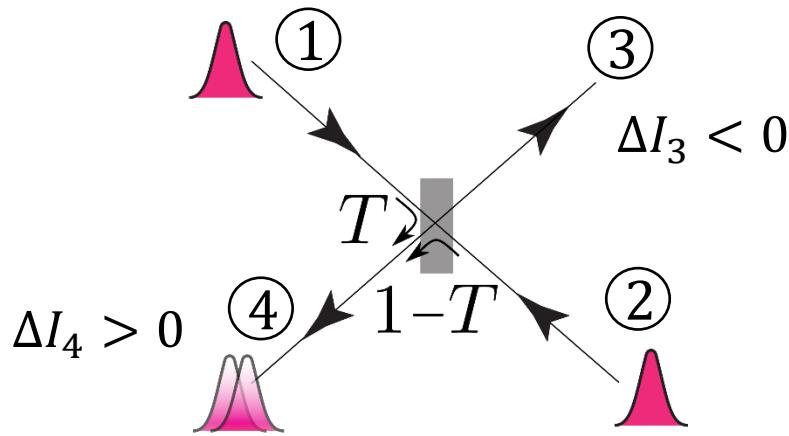


$T \ll 1$ : processus de Poisson

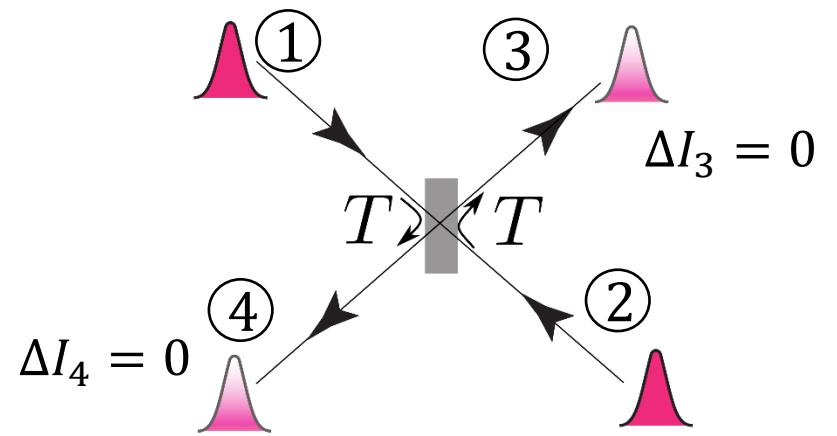
$$\langle \Delta N_T^2 \rangle = \langle N_T \rangle = TN_0$$

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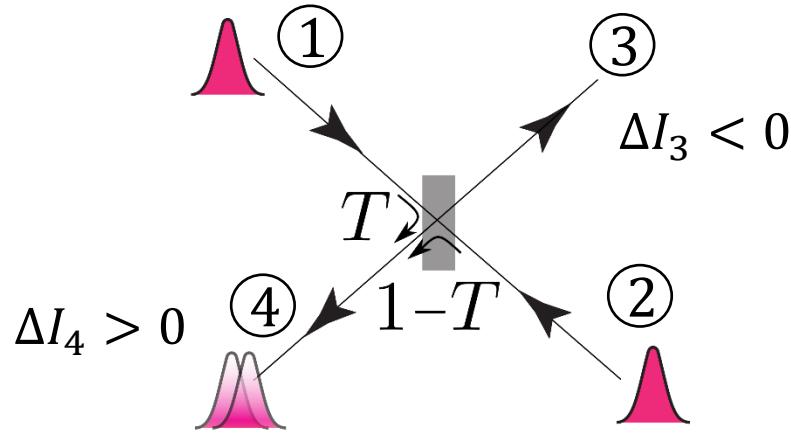




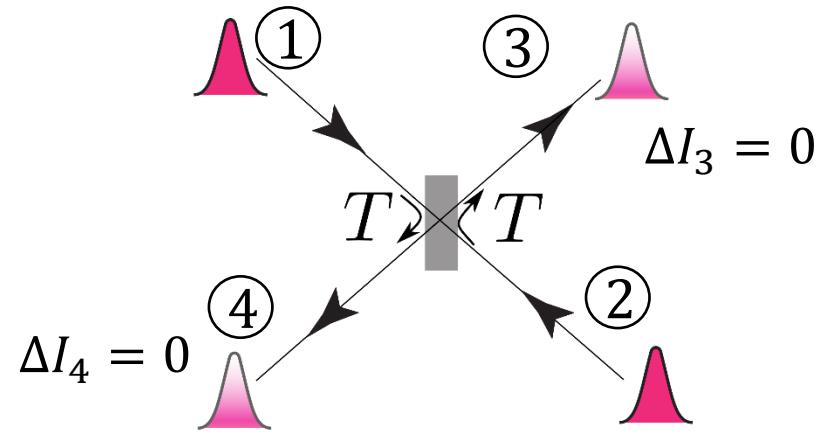
Groupement  
 $\langle \Delta I_3 \Delta I_4 \rangle < 0$



Dégroupement  
 $\langle \Delta I_3 \Delta I_4 \rangle = 0$

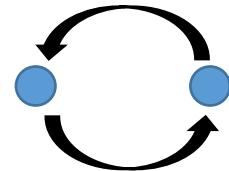


Groupement  
 $\langle \Delta I_3 \Delta I_4 \rangle < 0$



Dégroupement  
 $\langle \Delta I_3 \Delta I_4 \rangle = 0$

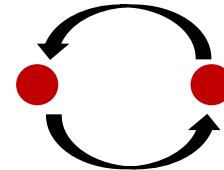
Anyons



$$\varphi = \pi/3 \ (\nu = 1/3)$$

Peuvent se regrouper en paquets  
 $\nu = 1/3$ , 3 anyons pour 1 état inaccessible  
 Haldane PRL 67 937 (1991)

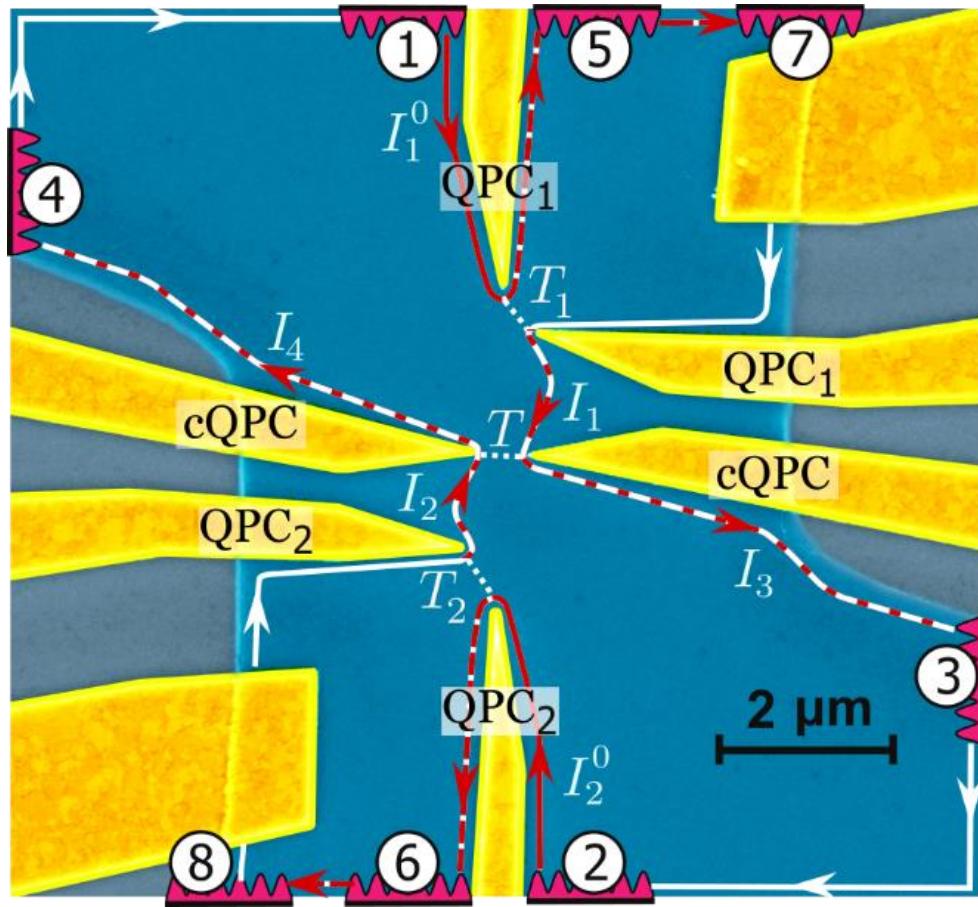
Electrons: fermions



$$\varphi = \pi$$

Dégroupement complet  
 Principe d'exclusion de Pauli

# Le collisionneur à anyons



Particules émises dans les bras d'entrées avec une probabilité:  $T_1 = T_2 = T_S$

Courant total  $I_+ = I_1 + I_2$

Différence de courant  $I_- = I_1 - I_2 = 0$

Pseudo-facteur de Fano

$$\langle \Delta I_3 \Delta I_4 \rangle = P 2qT(1 - T)I_+/T_{meas}$$

Prédictions:

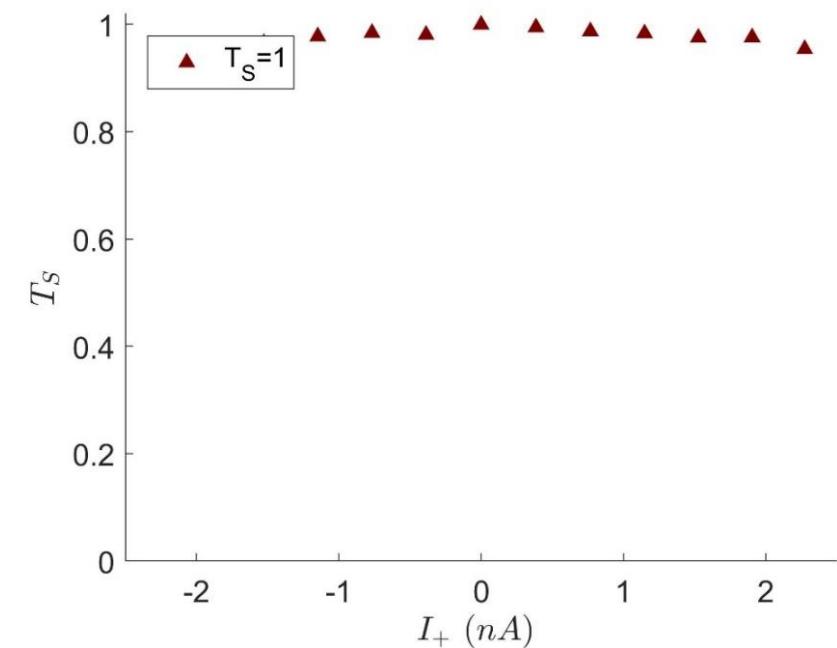
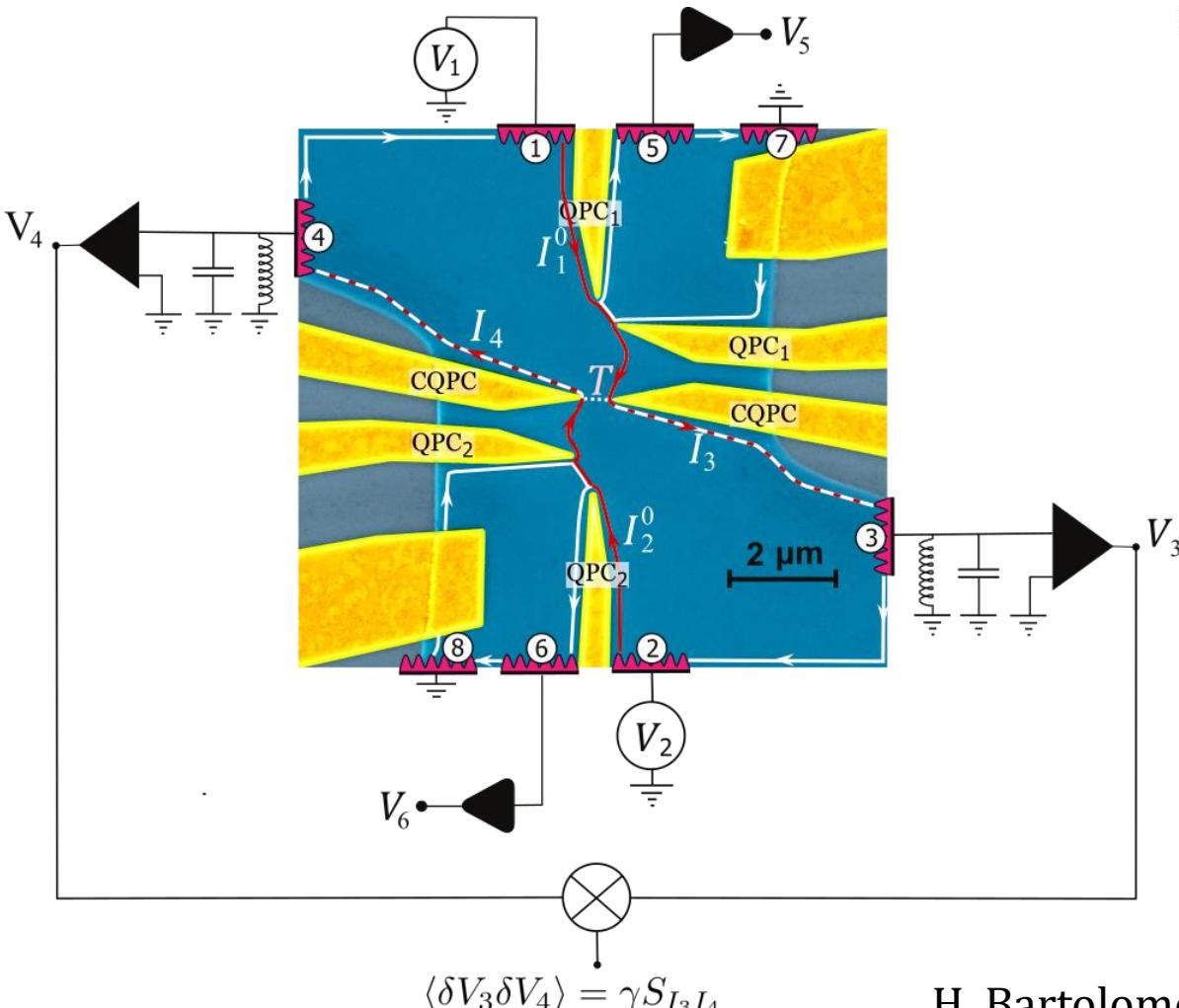
Electrons (fermions),  $P = 0$

Anyons ( $v = 1/3$ ),  $P = -2$  (for  $\varphi = \frac{\pi}{3}$ )

B. Rosenow et al., PRL 116, 156802 (2016)

Cas entier:

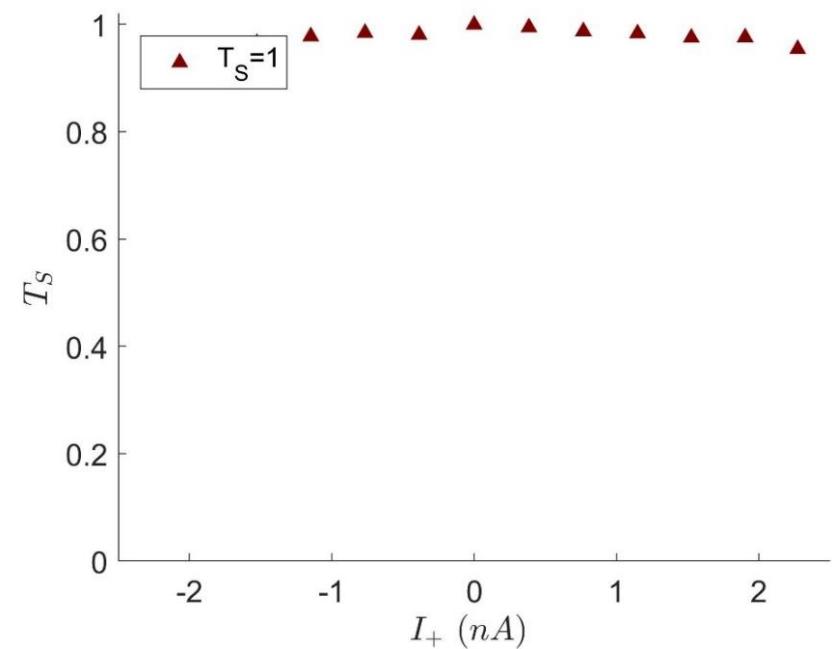
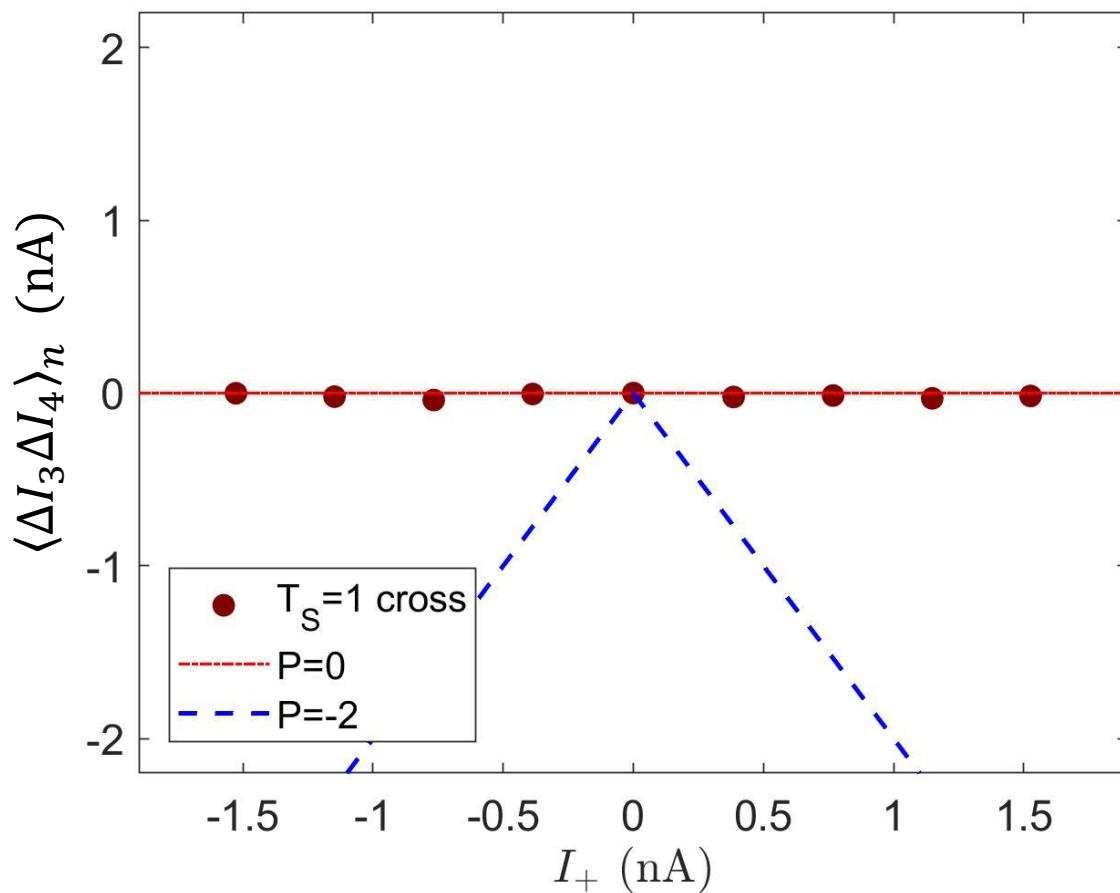
$$\nu = 2, T = 0.4, T_S = 1$$



# Collisions d'électrons, $\nu = 2$

Cas entier:

$$\nu = 2, T = 0.4, T_S = 1$$



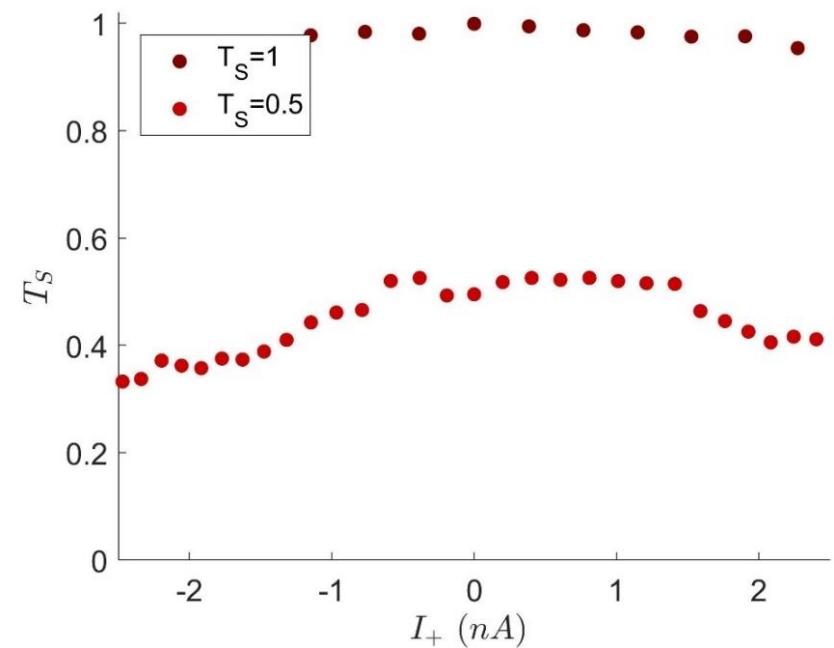
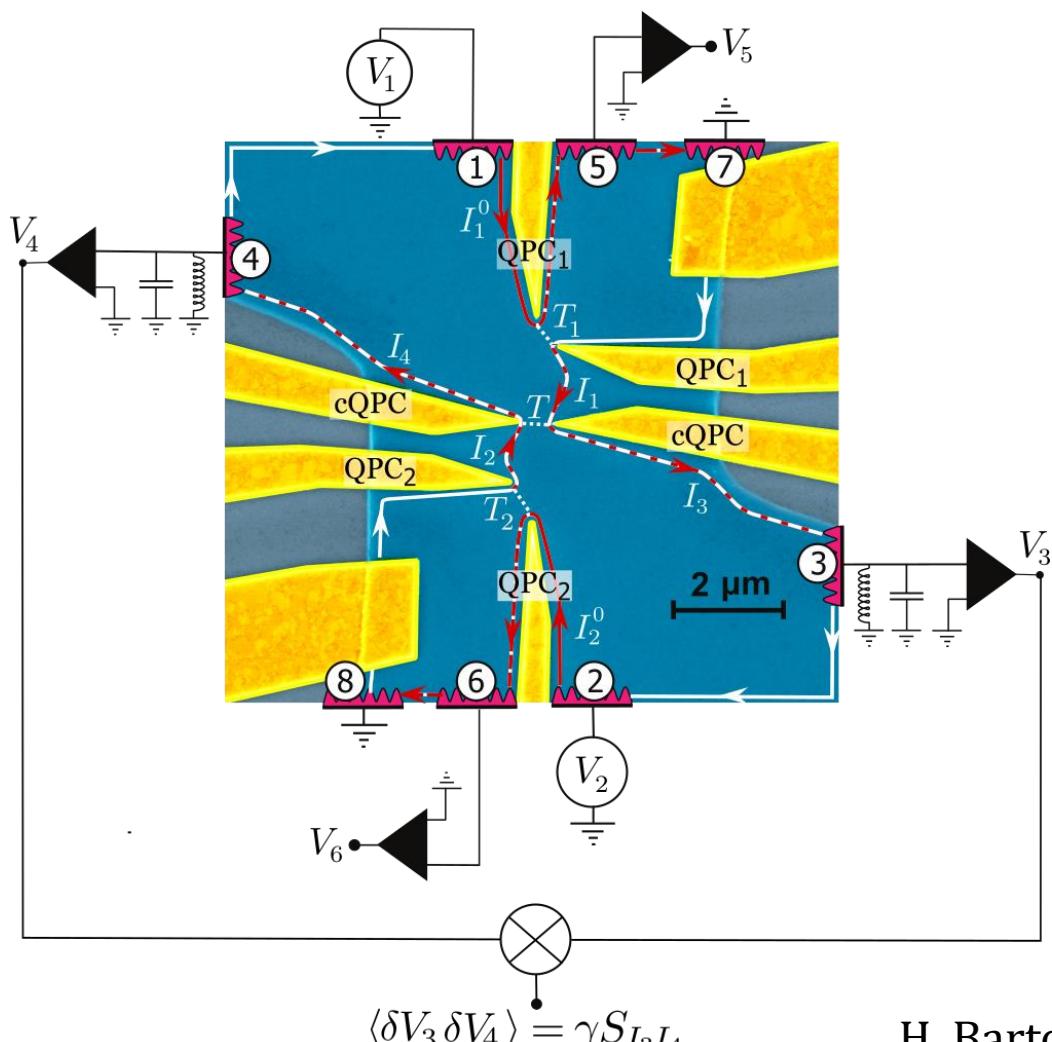
$$\langle \Delta I_3 \Delta I_4 \rangle_n = \frac{\langle \Delta I_3 \Delta I_4 \rangle}{2qT(1-T)/T_{meas}}$$

$$\langle \Delta I_3 \Delta I_4 \rangle_n = PI_+$$

# Collisions d'électrons, $\nu = 2$

Cas entier:

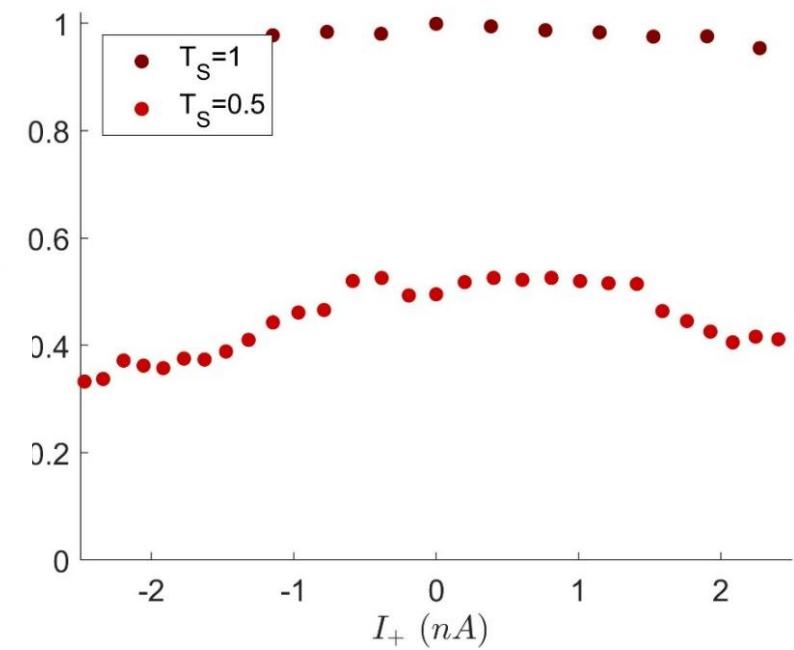
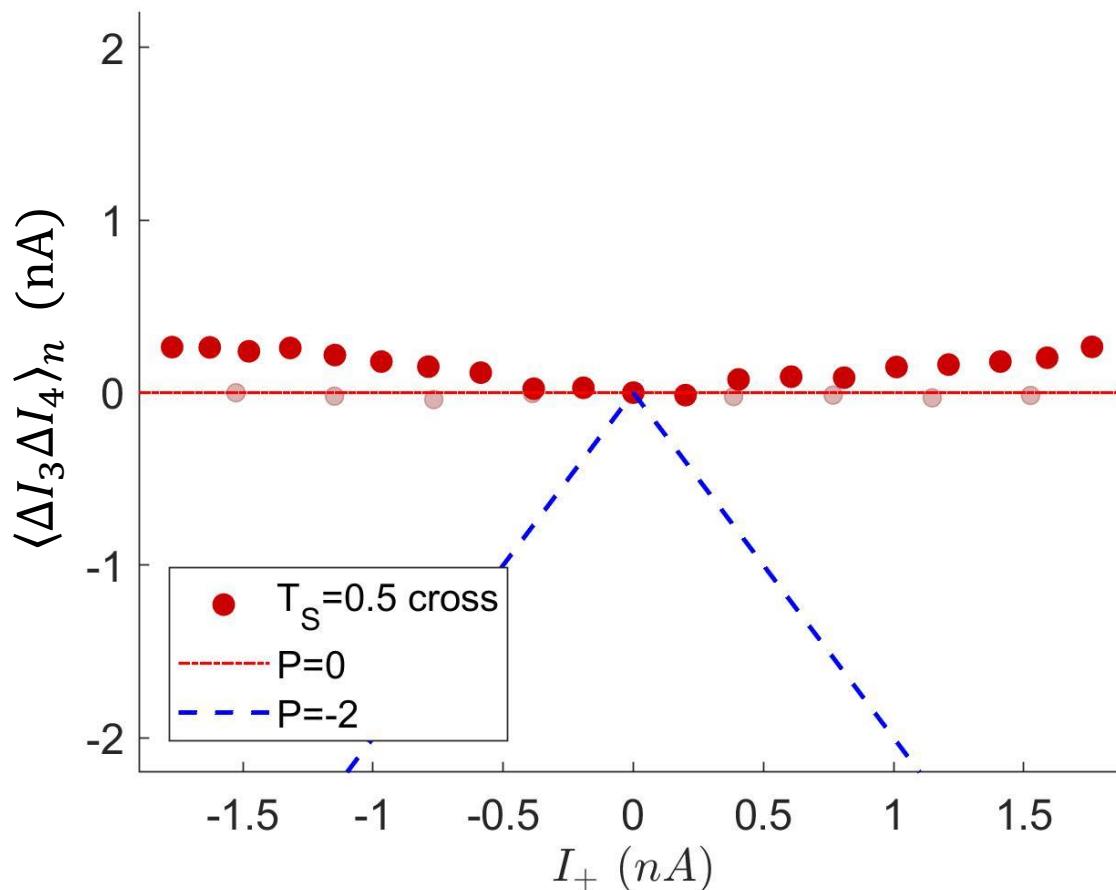
$$\nu = 2, T = 0.4, T_S = 0.5$$



# Collisions d'électrons, $\nu = 2$

Cas entier:

$\nu = 2, T = 0.4, T_S = 0.5$



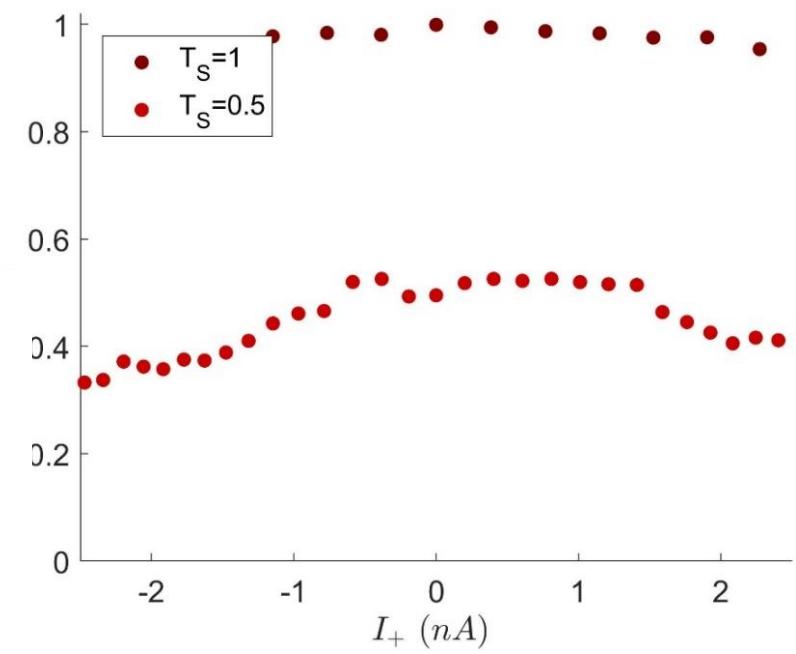
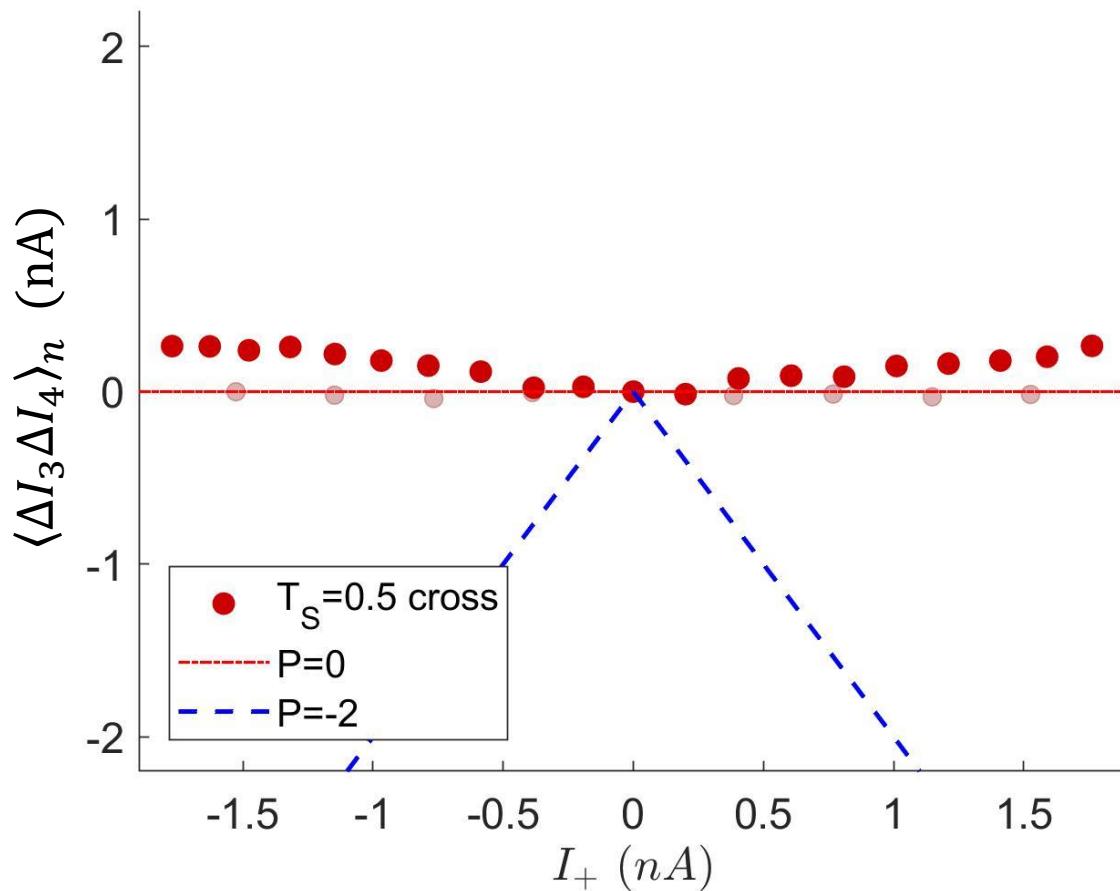
$$\langle \Delta I_3 \Delta I_4 \rangle_n = \frac{\langle \Delta I_3 \Delta I_4 \rangle}{2qT(1-T)/T_{meas}}$$

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# Collisions d'électrons, $\nu = 2$

Cas entier:

$\nu = 2, T = 0.4, T_S = 0.5$



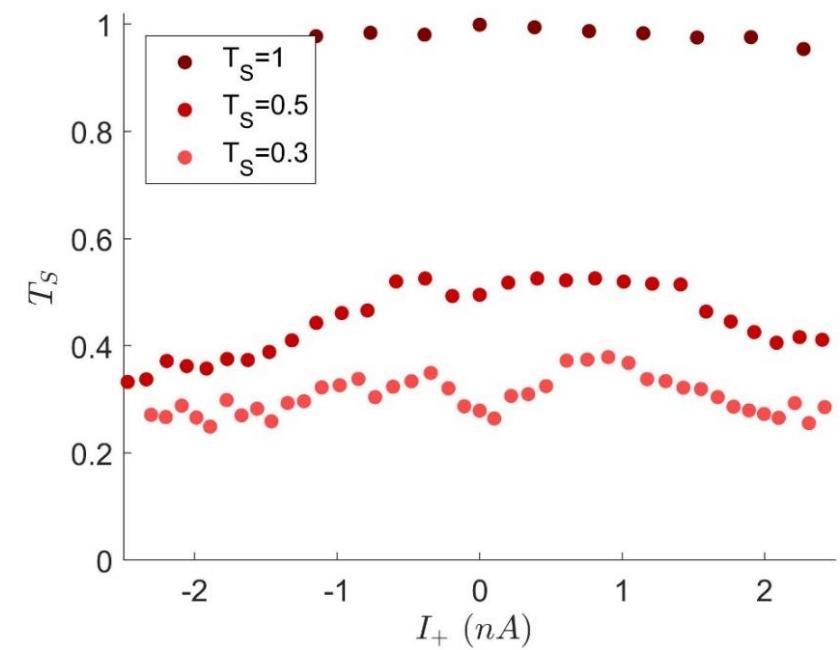
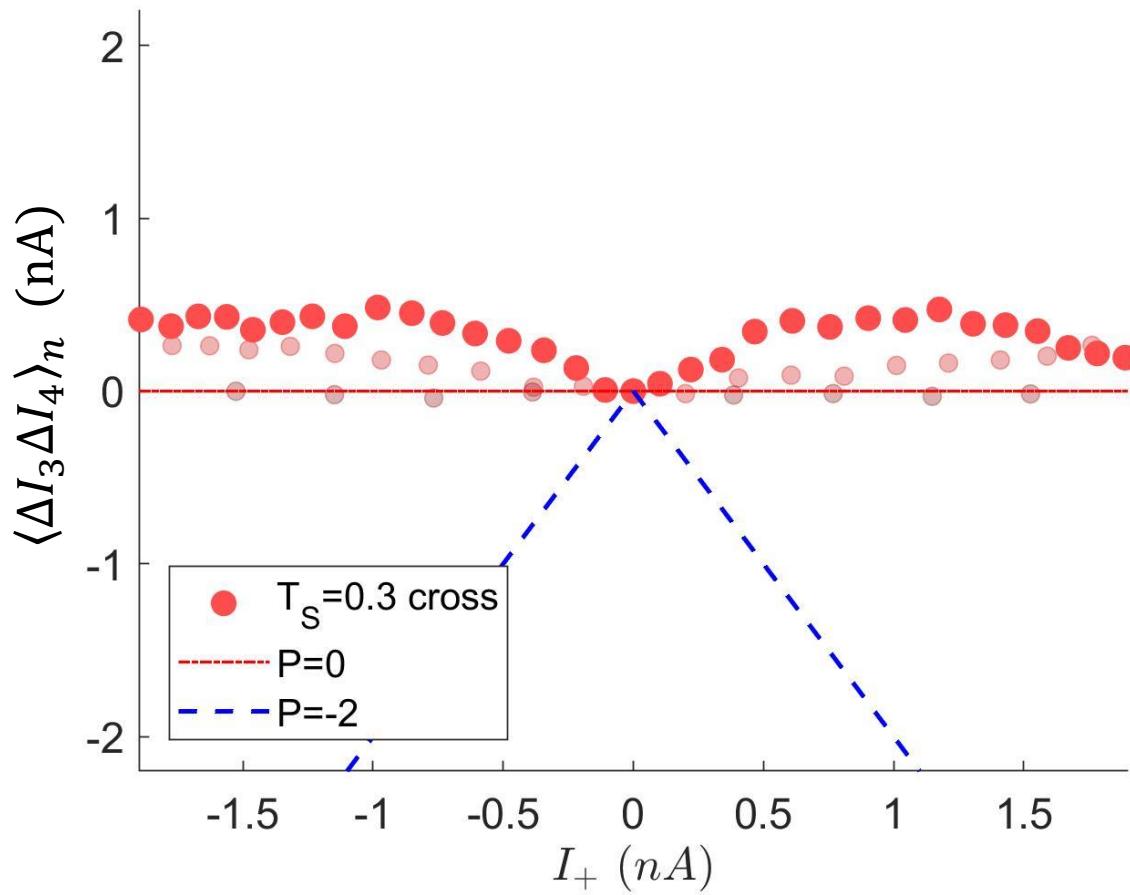
$$\langle \Delta I_3 \Delta I_4 \rangle_n = \frac{\langle \Delta I_3 \Delta I_4 \rangle}{2qT(1-T)/T_{meas}}$$

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# Collisions d'électrons, $\nu = 2$

Cas entier:

$$\nu = 2, T = 0.4, T_S = 0.5$$



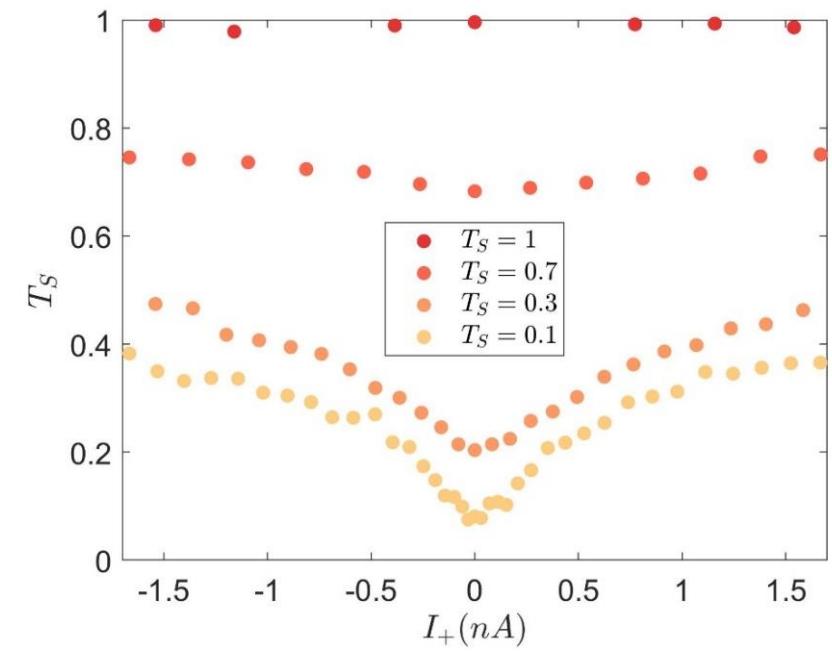
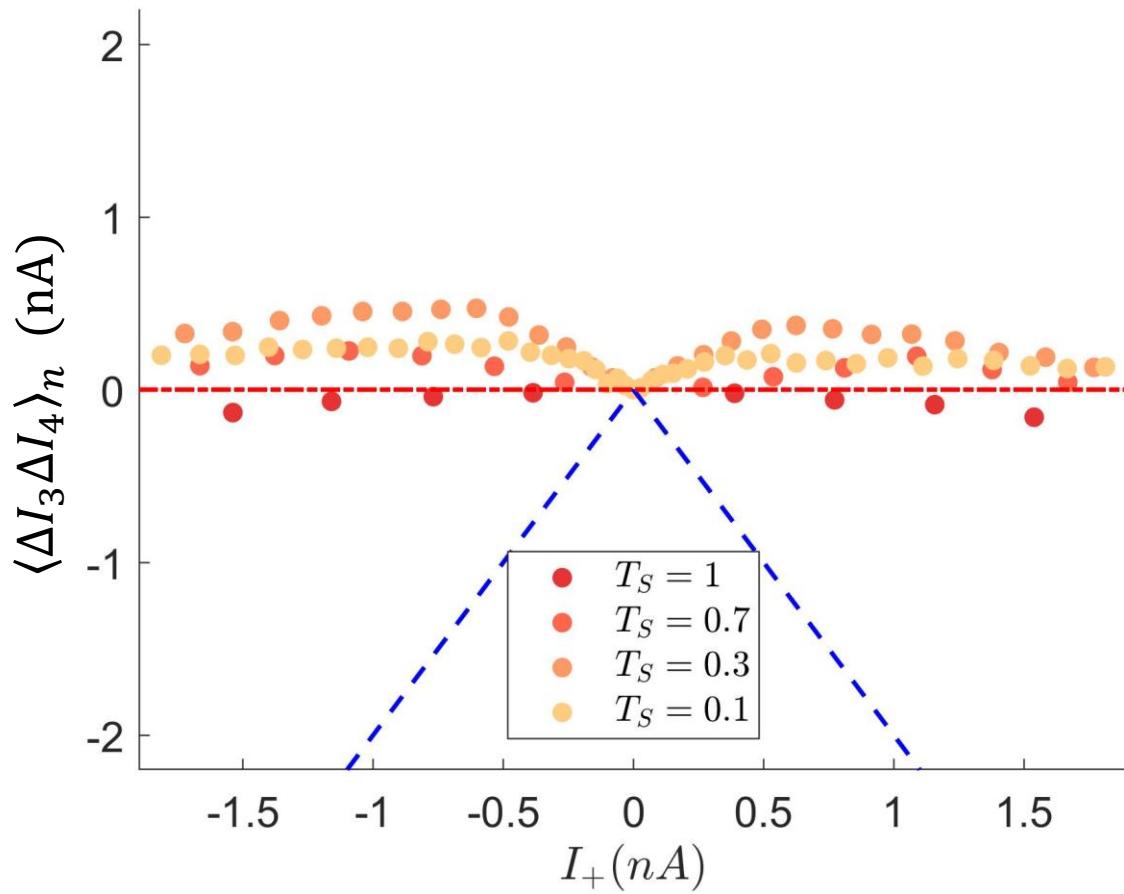
$$\langle \Delta I_3 \Delta I_4 \rangle_n = \frac{\langle \Delta I_3 \Delta I_4 \rangle}{2qT(1-T)/T_{meas}}$$

$$\langle \Delta I_3 \Delta I_4 \rangle_n = PI_+$$

# Collisions d'électrons, $\nu = 3$

Cas entier:

$\nu = 3, T = 0.5$



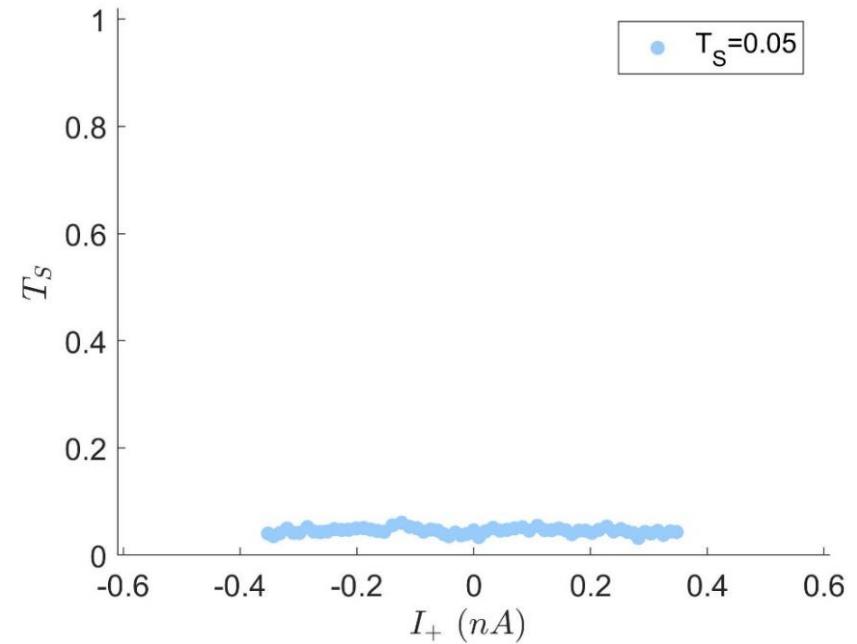
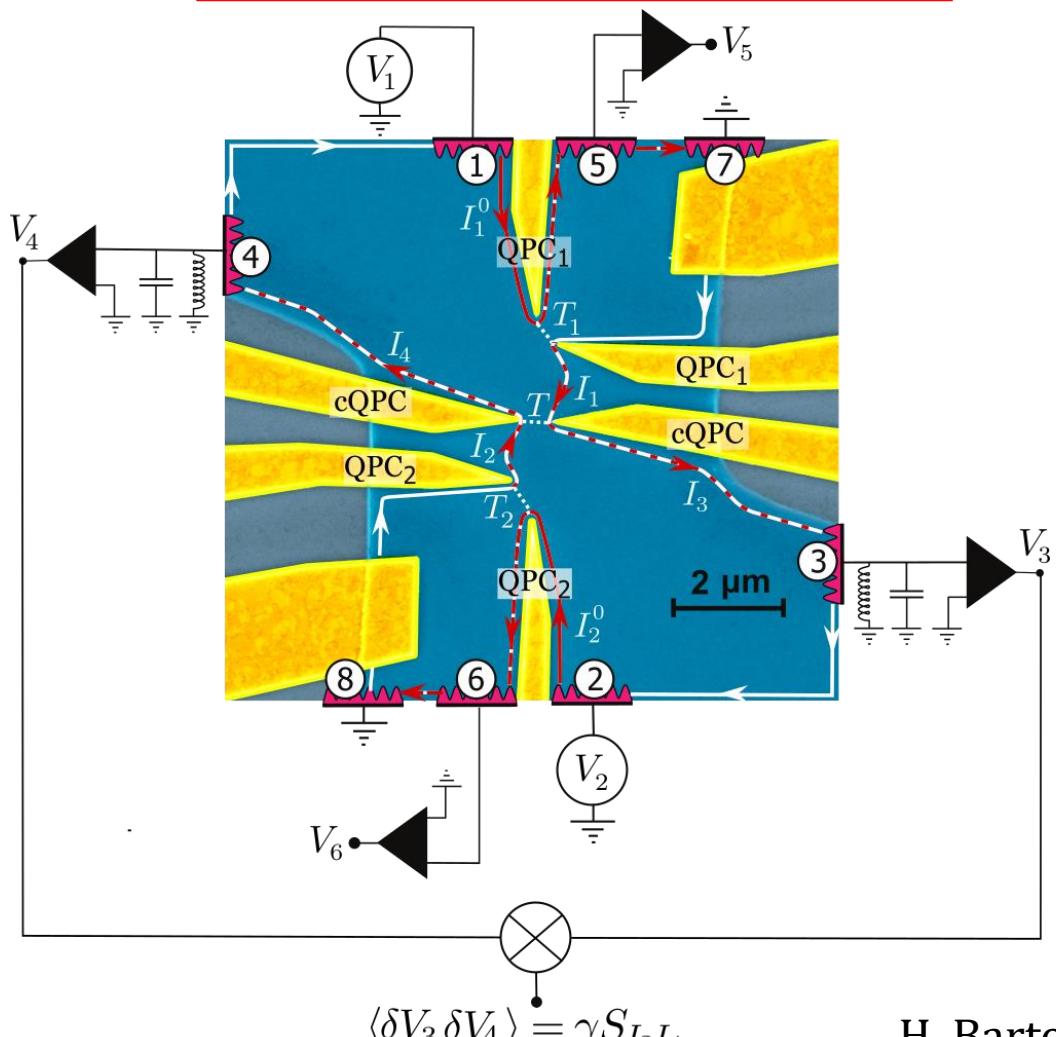
$$\langle \Delta I_3 \Delta I_4 \rangle_n = \frac{\langle \Delta I_3 \Delta I_4 \rangle}{2qT(1-T)/T_{meas}}$$

$$\langle \Delta I_3 \Delta I_4 \rangle_n = PI_+$$

# Collisions d'anyons, $\nu = 1/3$

Cas fractionnaire:

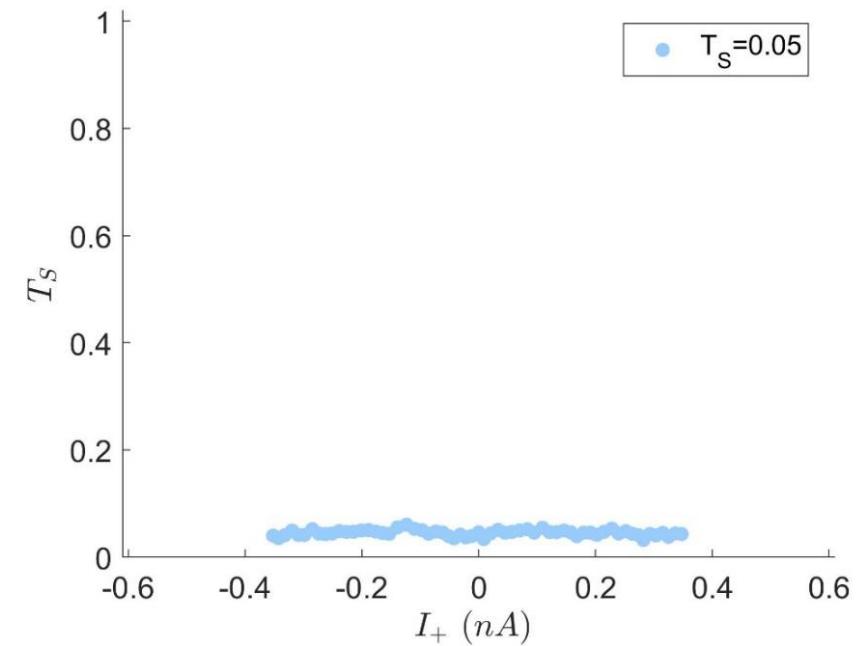
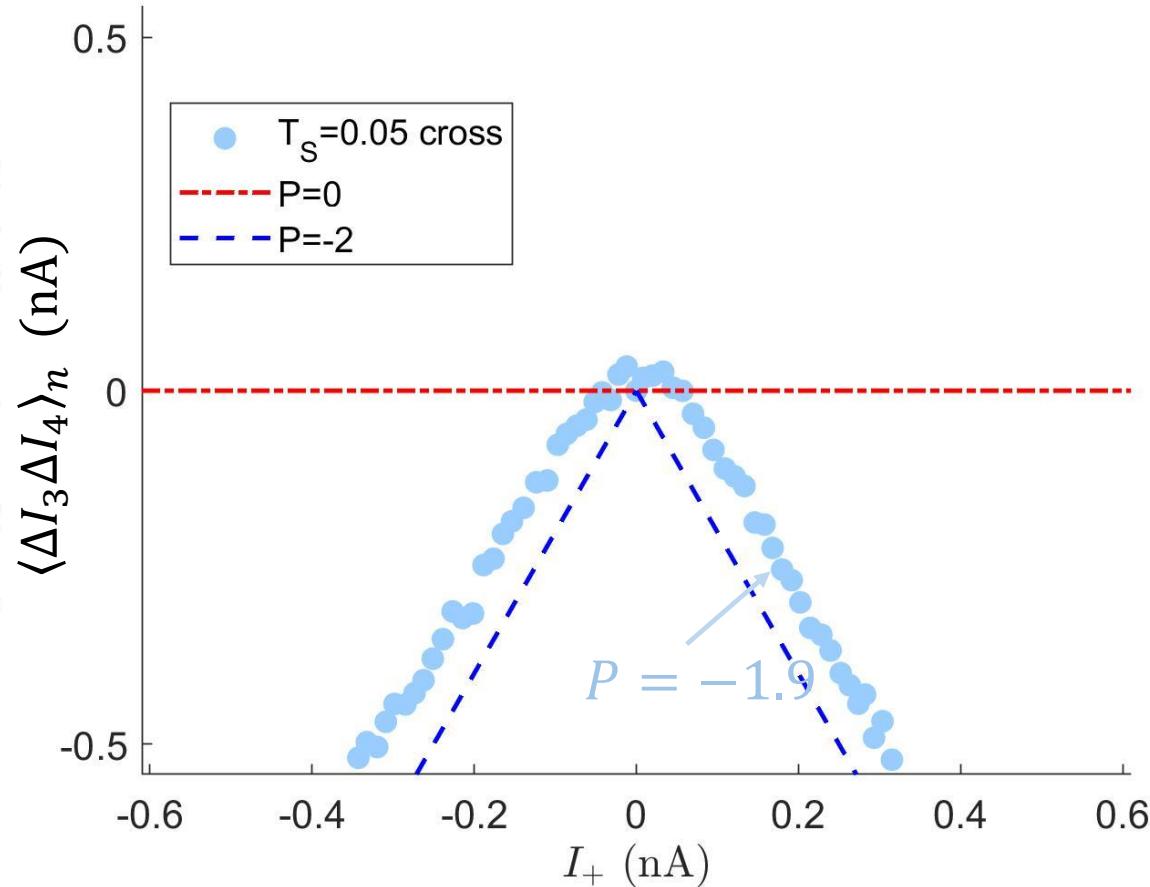
$$\nu = \frac{1}{3}, T = 0.3, T_S = 0.05 \ll 1$$



# Collisions d'anyons, $\nu = 1/3$

Cas fractionnaire:

$$\nu = \frac{1}{3}, T = 0.3, T_S = 0.05 \ll 1$$



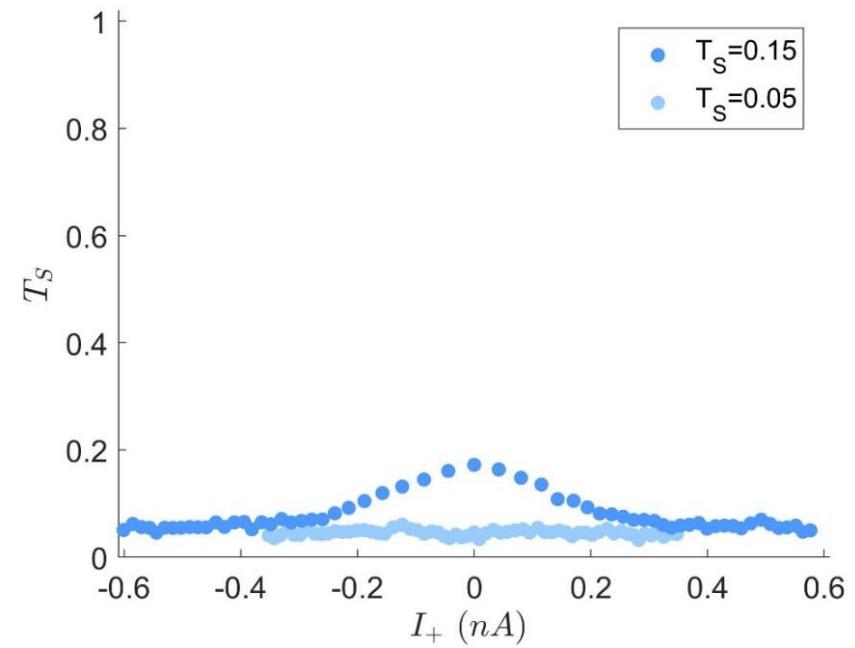
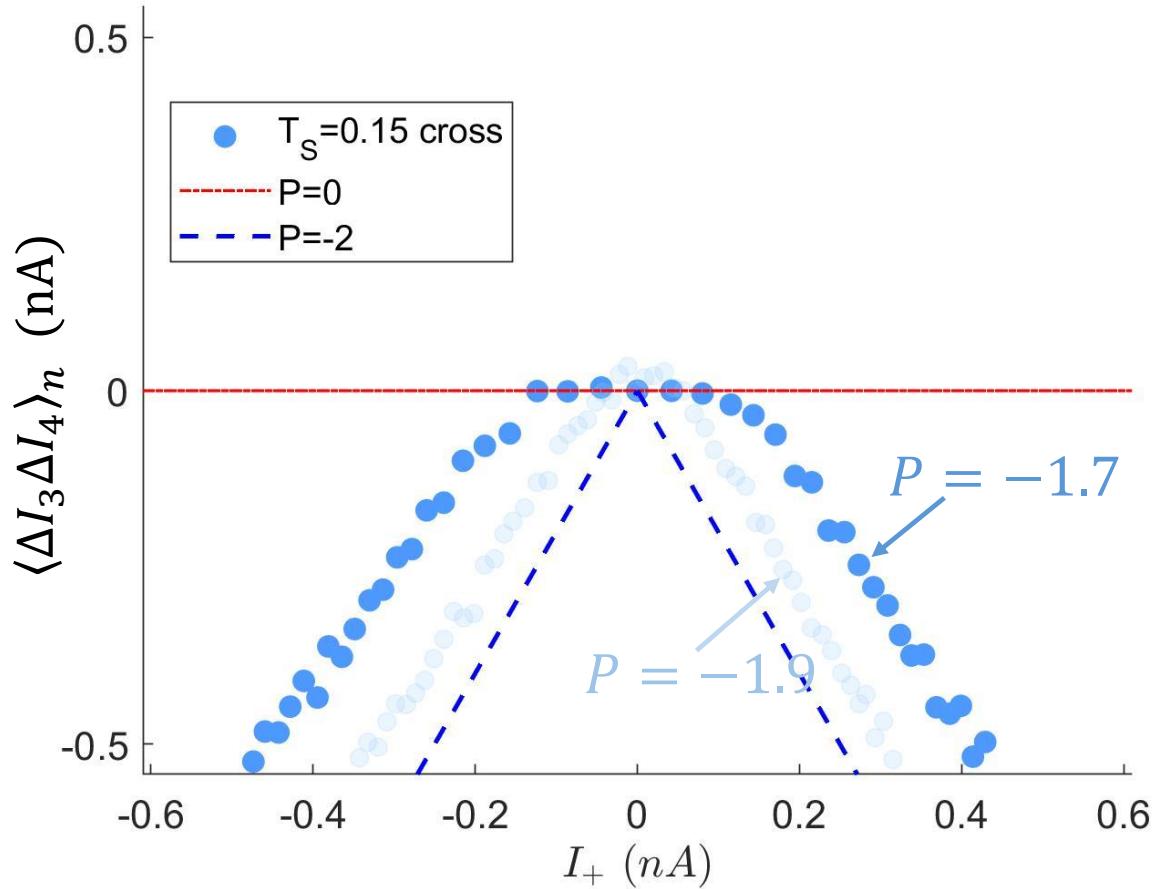
$$\langle \Delta I_3 \Delta I_4 \rangle_n = \frac{\langle \Delta I_3 \Delta I_4 \rangle}{2qT(1-T)/T_{meas}}$$

$$\langle \Delta I_3 \Delta I_4 \rangle_n = PI_+$$

# Collisions d'anyons, $\nu = 1/3$

Cas fractionnaire:

$$\nu = \frac{1}{3}, T = 0.3, T_S = 0.15$$



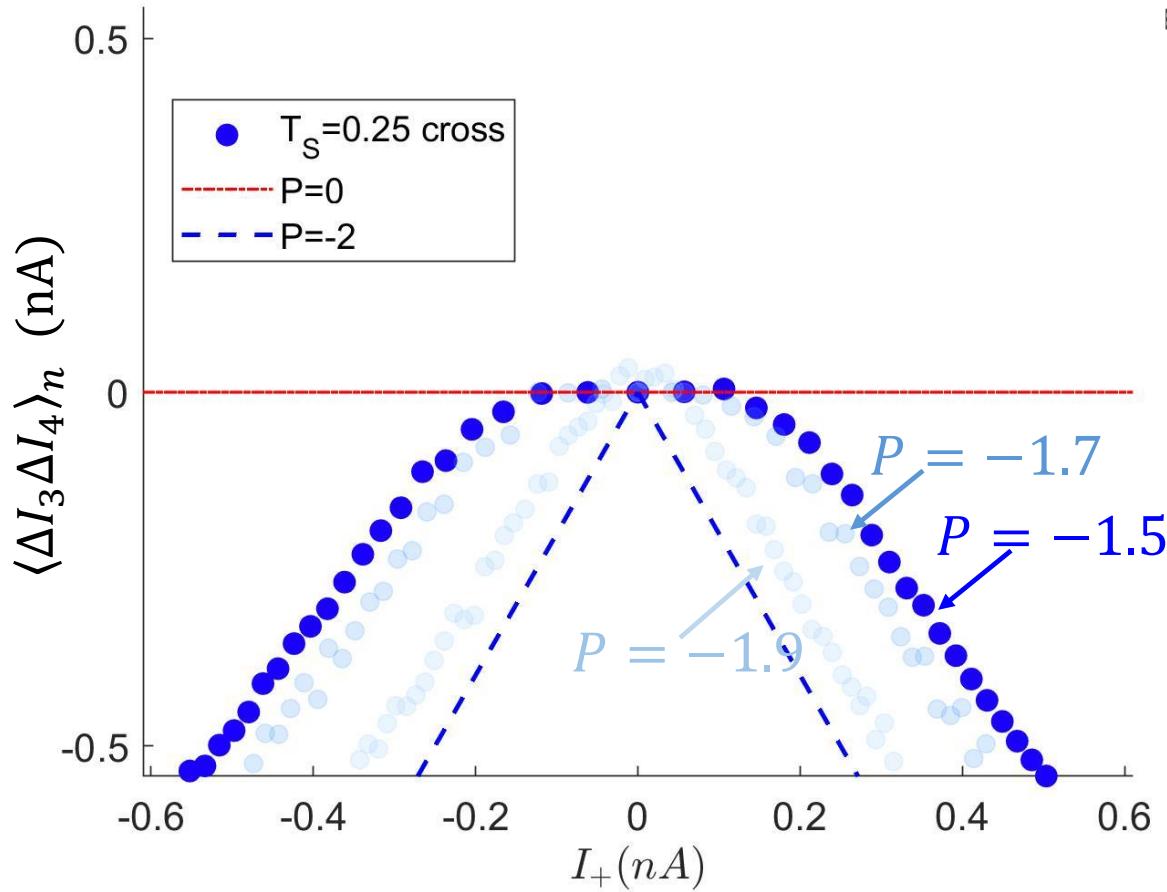
$$\langle \Delta I_3 \Delta I_4 \rangle_n = \frac{\langle \Delta I_3 \Delta I_4 \rangle}{2qT(1-T)/T_{meas}}$$

$$\langle \Delta I_3 \Delta I_4 \rangle_n = PI_+$$

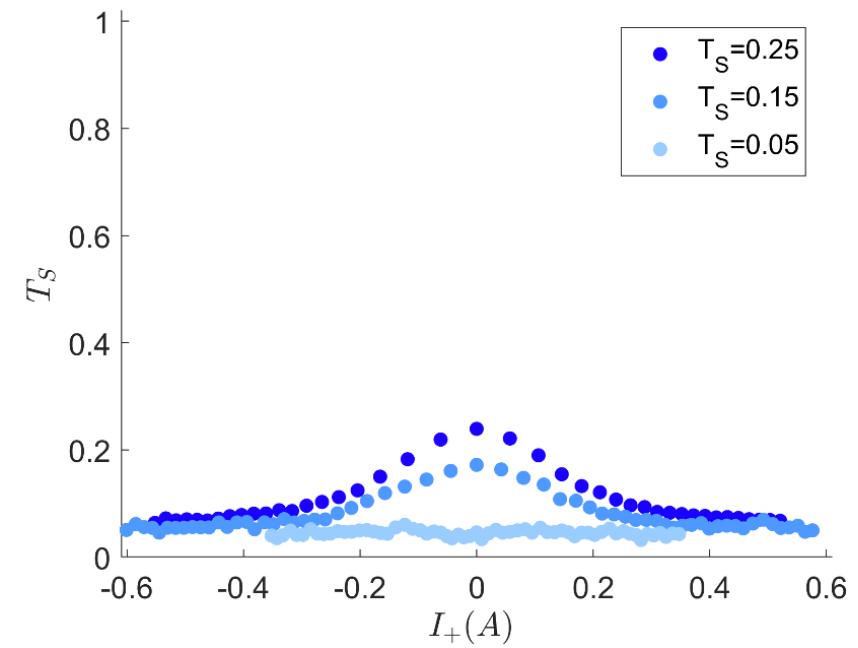
# Collisions d'anyons, $\nu = 1/3$

Cas fractionnaire:

$$\nu = \frac{1}{3}, T = 0.3, T_S = 0.25$$



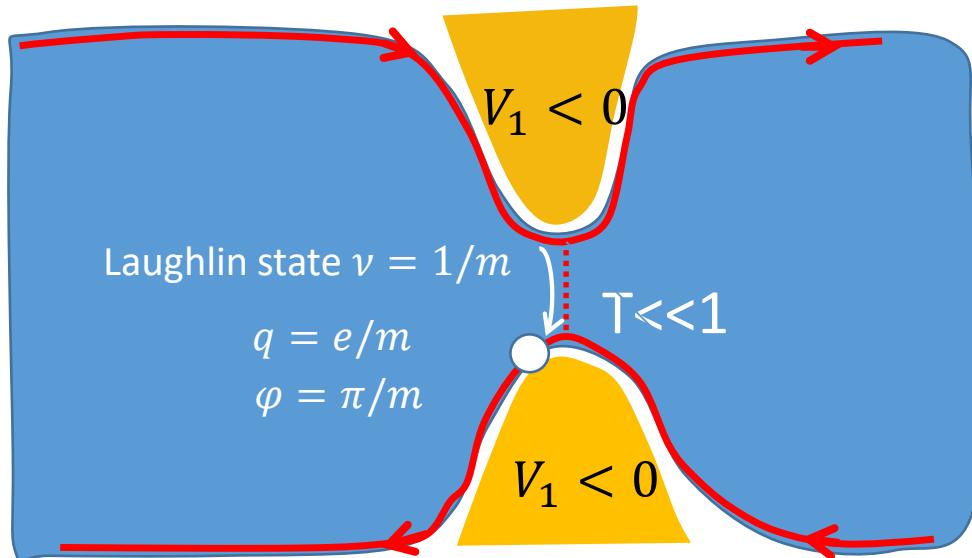
$P \approx -2$  anyon



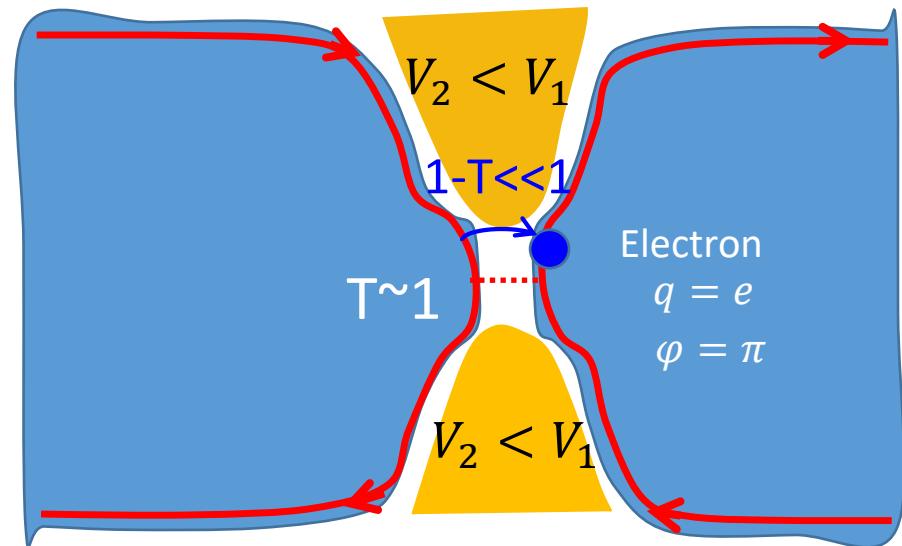
$$\langle \Delta I_3 \Delta I_4 \rangle_n = \frac{\langle \Delta I_3 \Delta I_4 \rangle}{2qT(1-T)/T_{meas}}$$

$$\langle \Delta I_3 \Delta I_4 \rangle_n = PI_+$$

Faible rétrodiffusion:  
 Transfert d'anyons



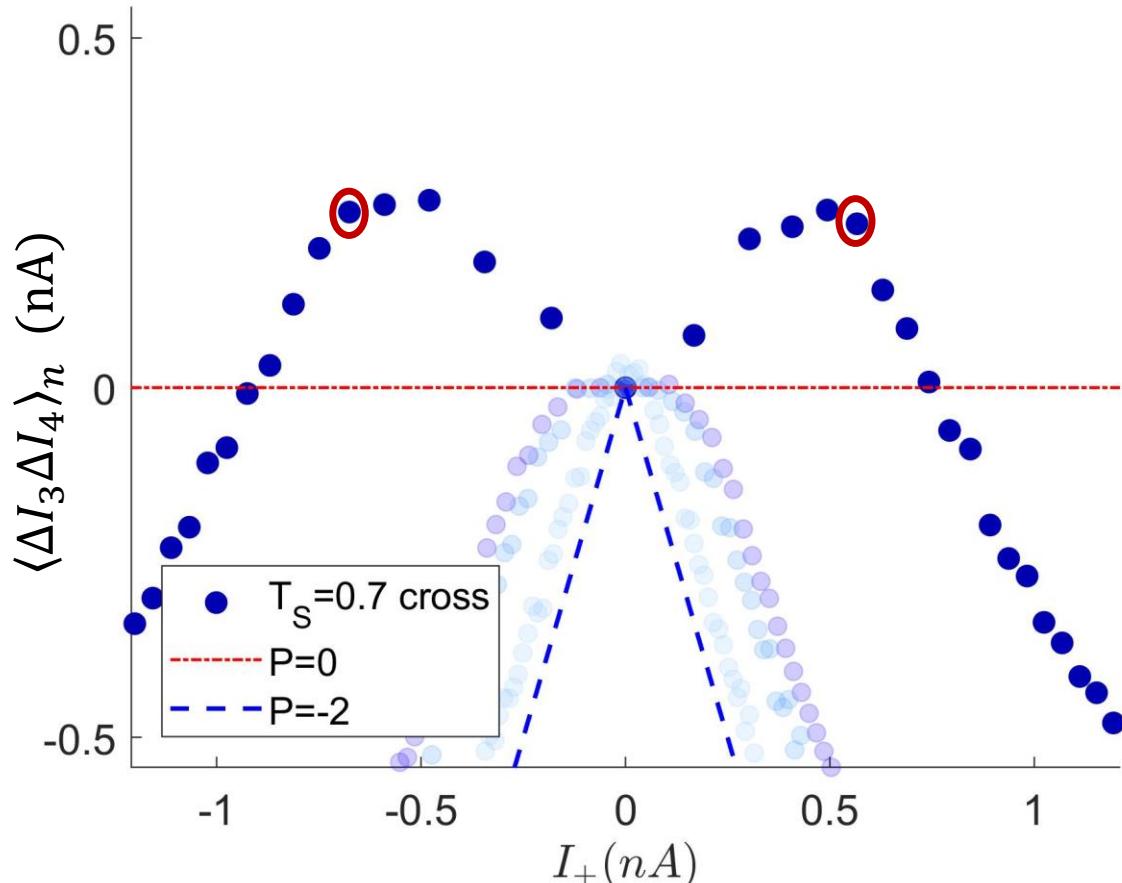
Forte rétrodiffusion:  
 Transfert d'électrons



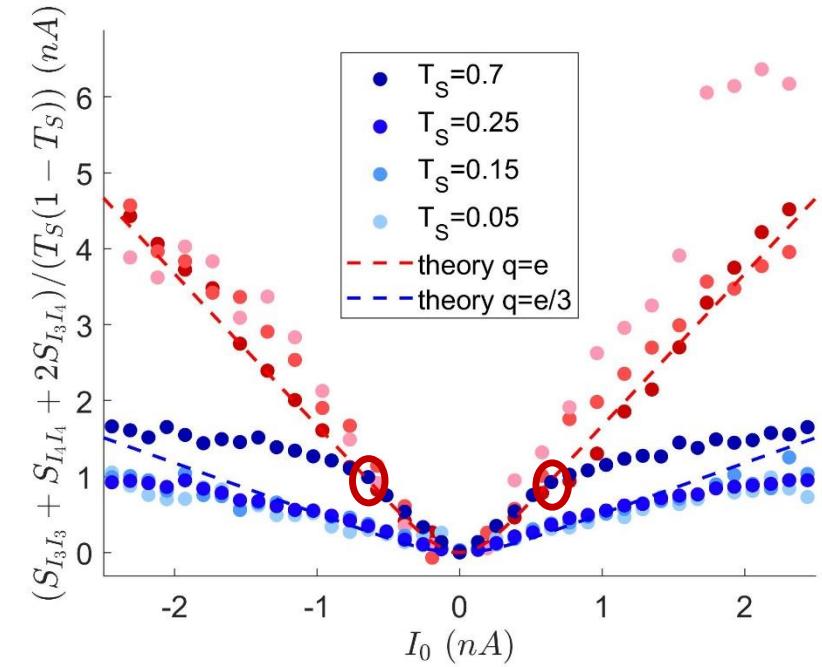
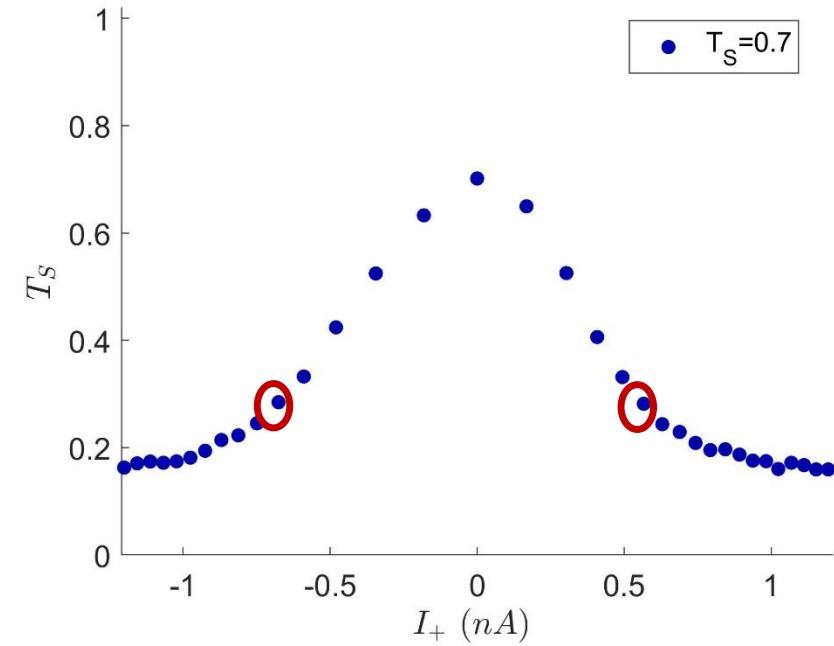
# Collision d'électrons/anyons à $\nu = 1/3$

Cas fractionnaire:

$$\nu = \frac{1}{3}, T = 0.3, T_S = 0.7$$

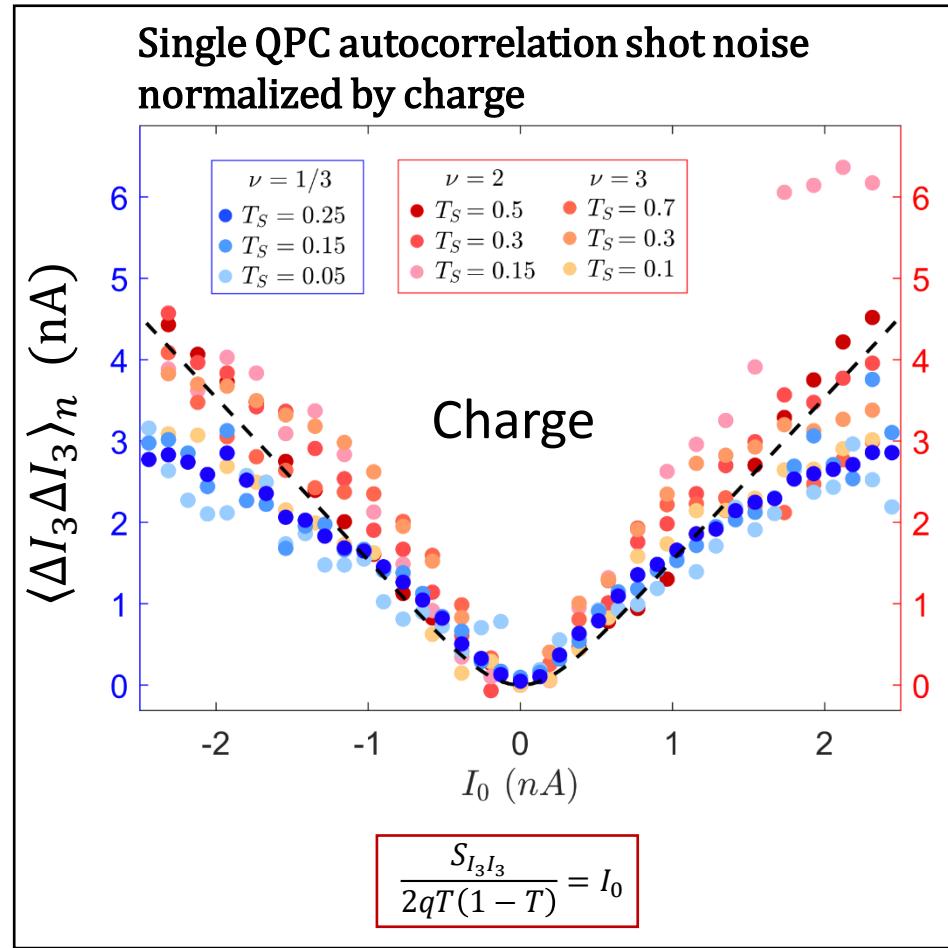


Strong backscattering, tunneling charge  $q=e$



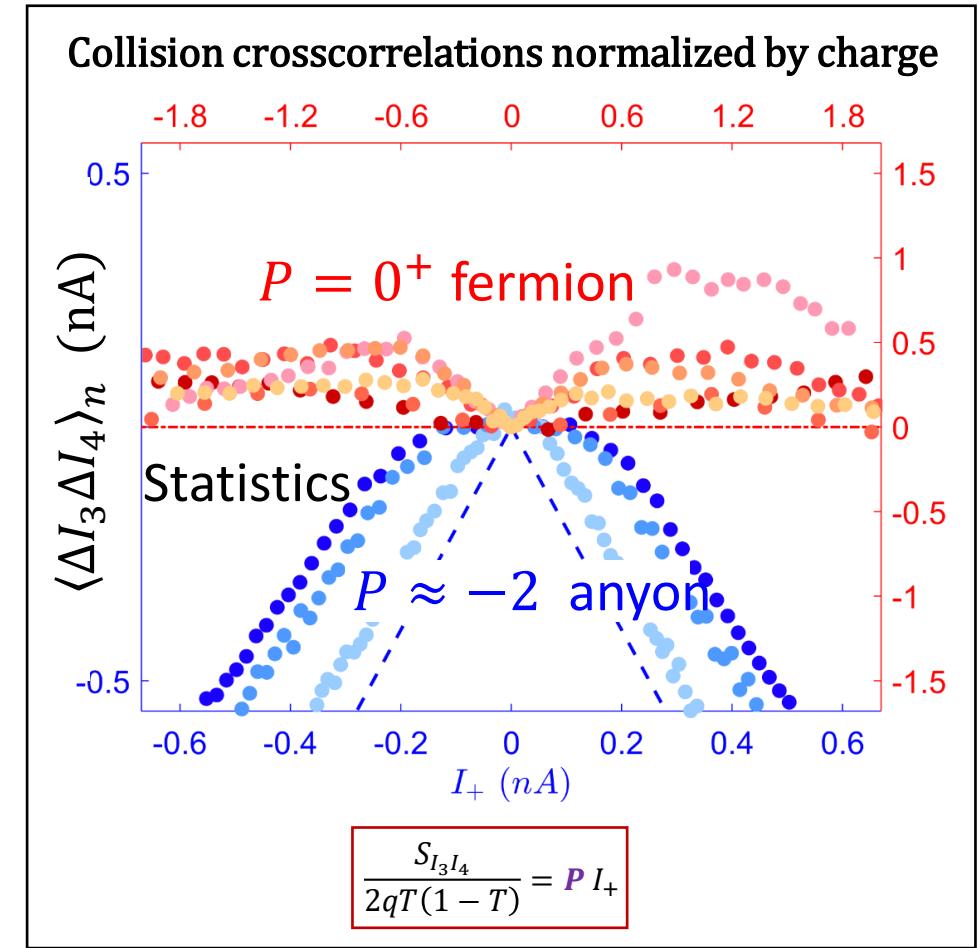
# Conclusion

- Colliders can be used to highlight fermionic/fractional statistics independently of charge



R. de Picciotto et al., Nature **389**, 162 (1997).

L. Saminadayar et al., Phys. Rev. Lett. **79**, 2526 (1997).



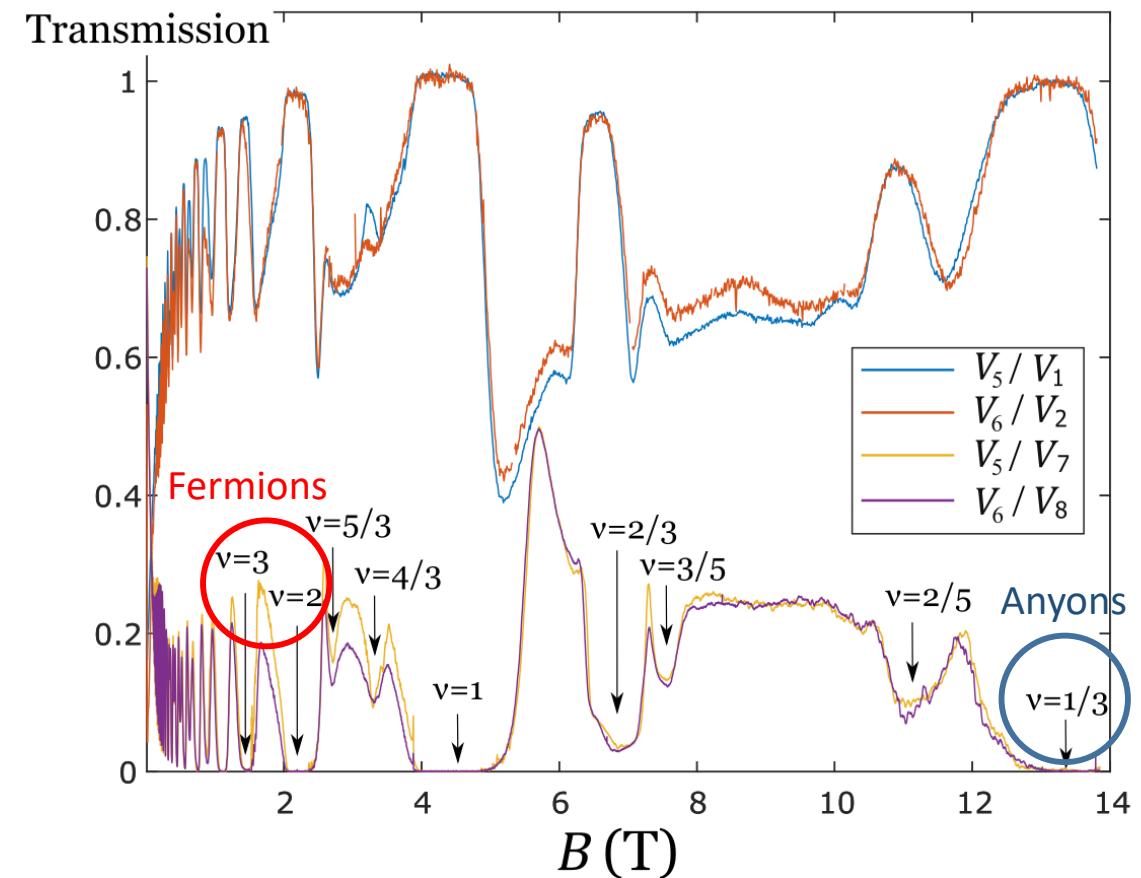
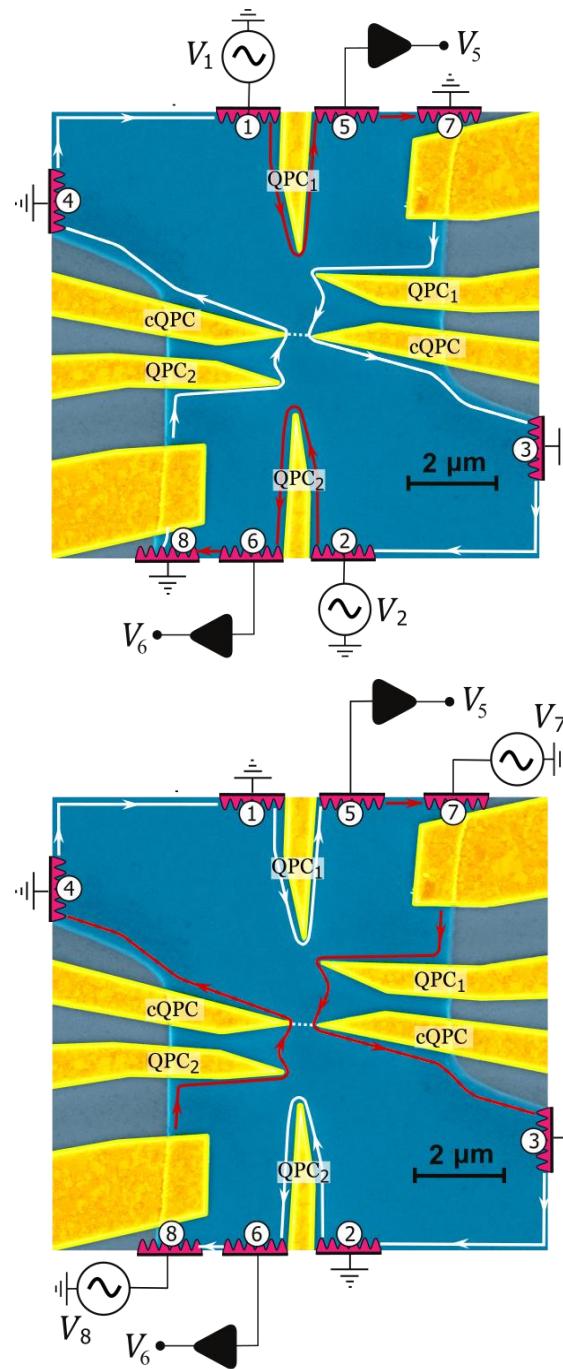
B. Rosenow, I. Levkivskyi and B. Halperin, PRL **116** 156802 (2016)

H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

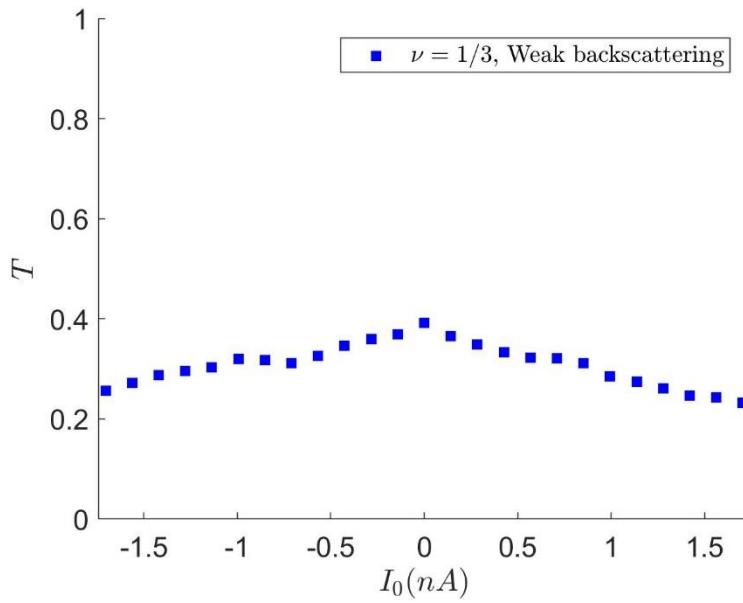
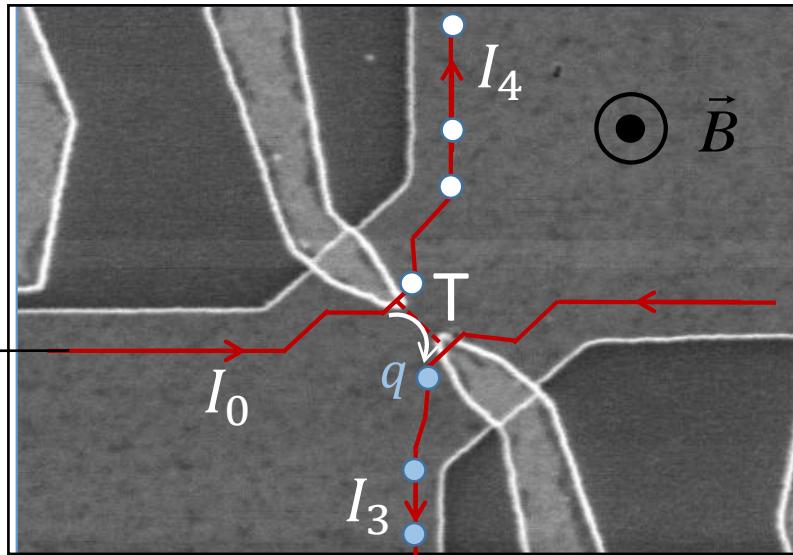
- Fractional statistics can also be measured in interferometers (Fabry-Perot)

J. Nakamura, S. Liang, G.C. Gardner, M.J. Manfra, Nature Physics **16** 931 (2020).

## The collider: sample

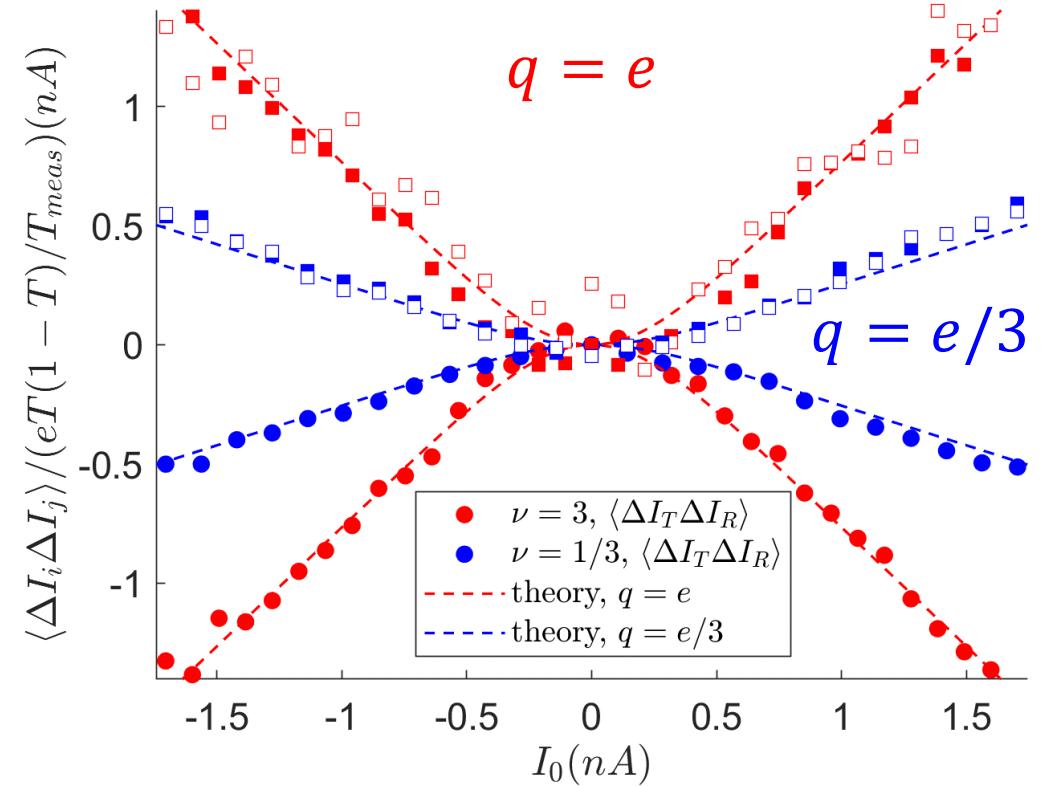


# Fractional charges and noise, $\nu=1/3$

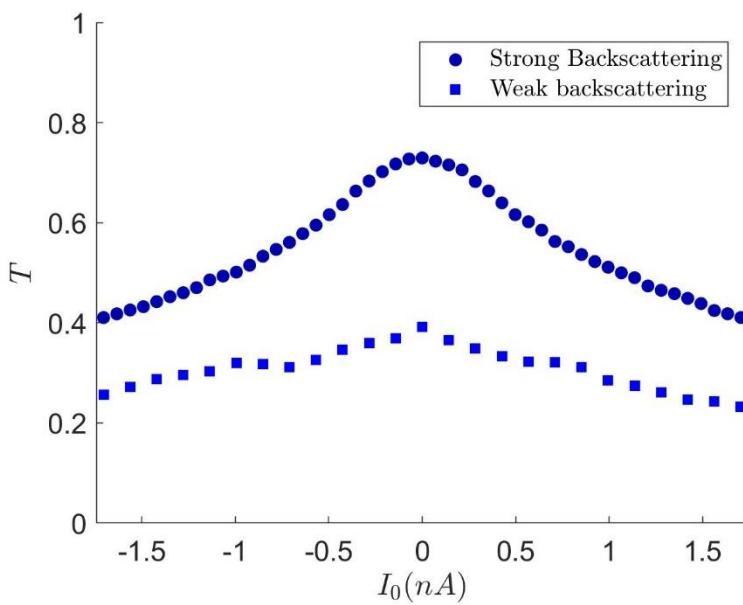
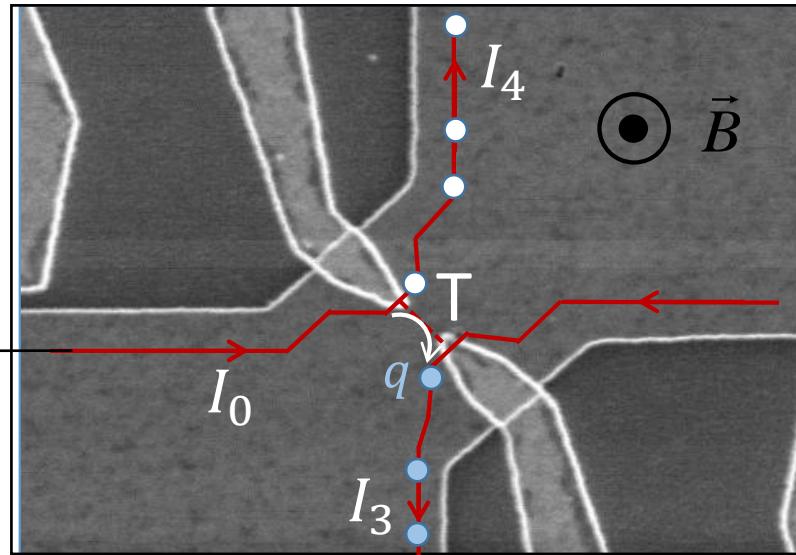


$T \ll 1$ : quasiparticle transfer is a poissonian process:  $\langle \Delta N_T^2 \rangle = \langle N_T \rangle = TN_0$

$$\langle \Delta I_T^2 \rangle = \frac{q^2}{T_{meas}^2} \langle \Delta N_T^2 \rangle = \frac{qT}{T_{meas}} \frac{qN_0}{T_{meas}} = \frac{qT}{T_{meas}} I_0$$

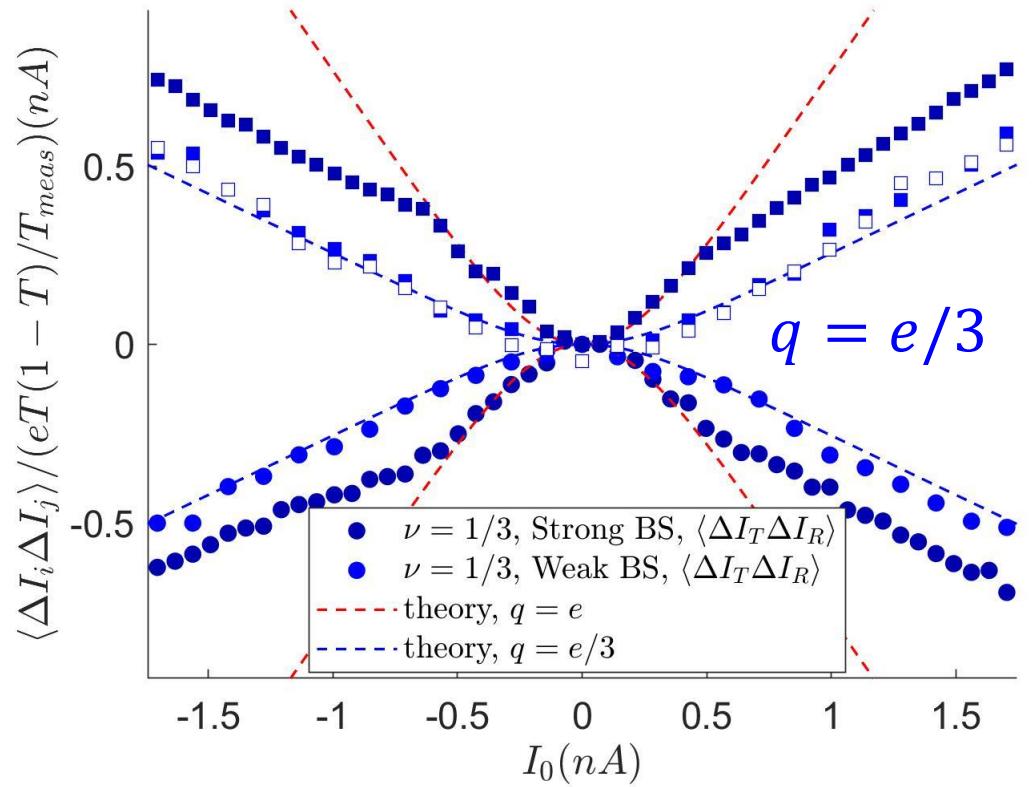


# Fractional charges and noise, $\nu=1/3$

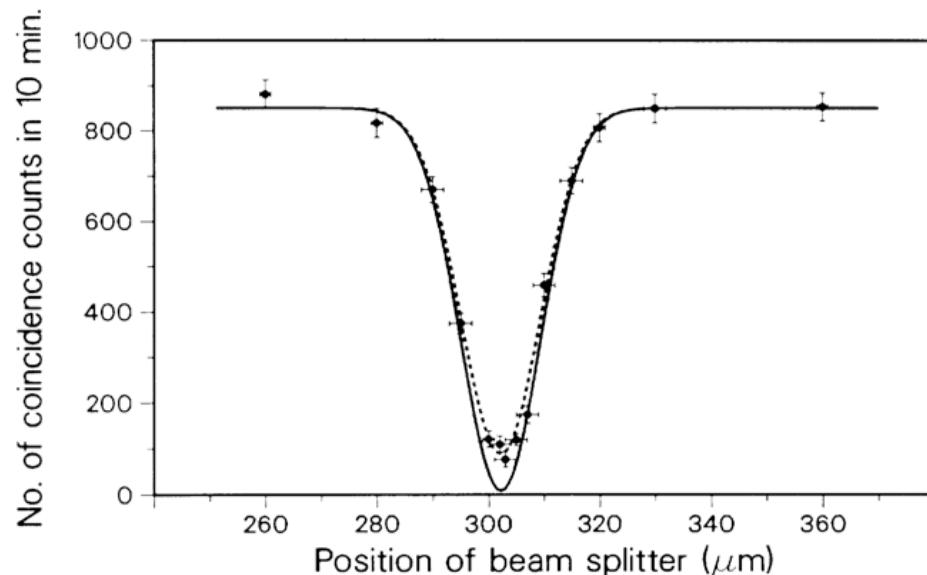
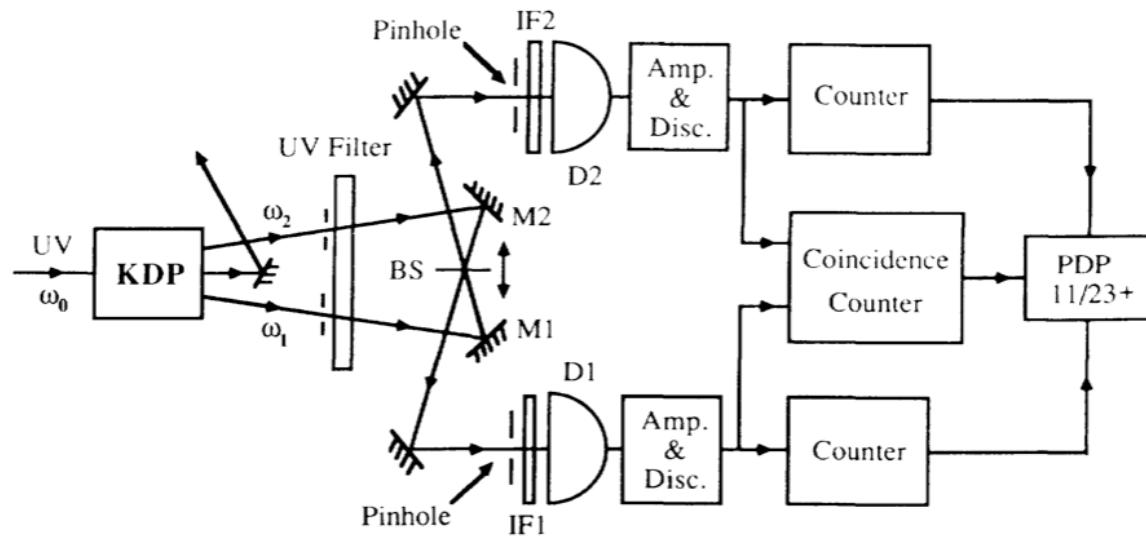


$T \ll 1$ : quasiparticle transfer is a poissonian process:  $\langle \Delta N_T^2 \rangle = \langle N_T \rangle = TN_0$

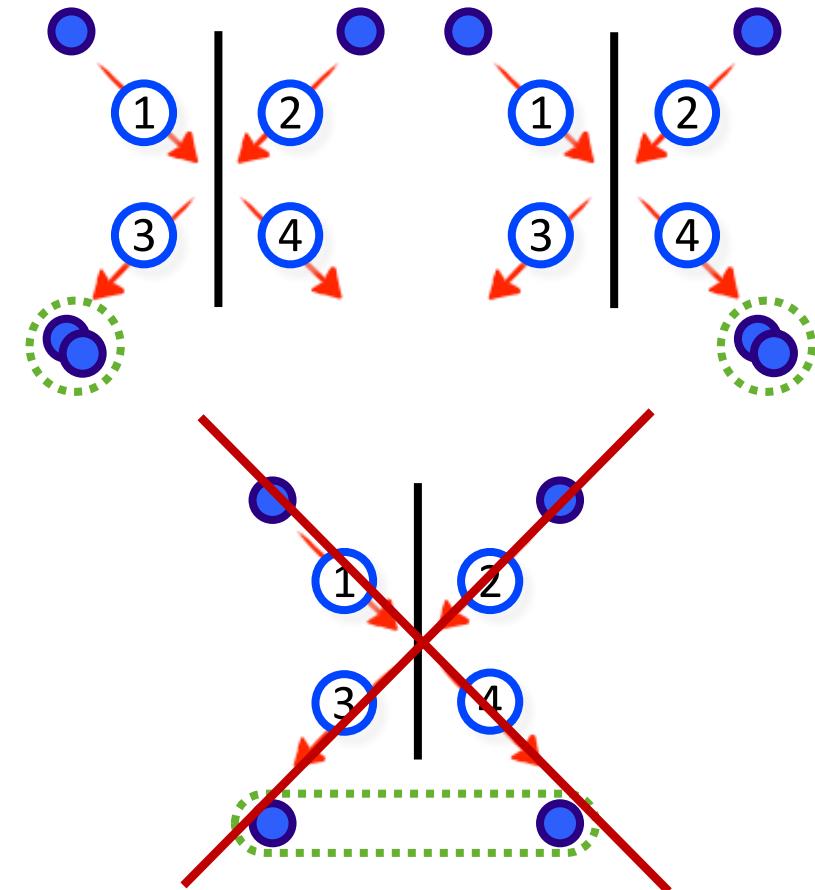
$$\langle \Delta I_T^2 \rangle = \frac{q^2}{T_{meas}^2} \langle \Delta N_T^2 \rangle = \frac{qT}{T_{meas}} \frac{qN_0}{T_{meas}} = \frac{qT}{T_{meas}} I_0$$



# Intéférences quantiques de 2 photons



Interférences quantiques  
à 2 particules



Photons pairs :

C. Hong *et al.*, PRL 59(18), 2044 (1987)

Different emitters :

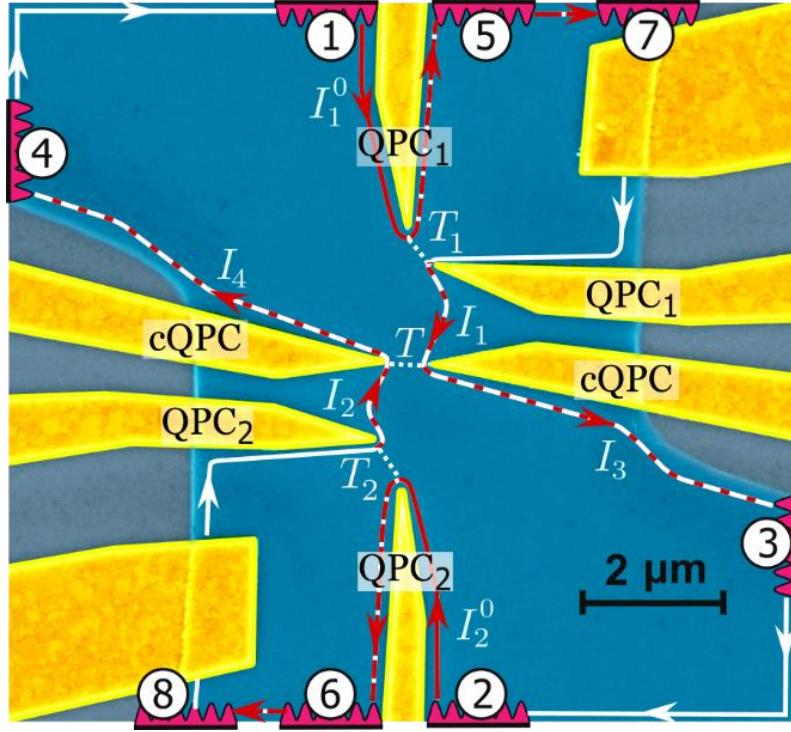
J. Beugnon *et al.*, Nature 440, 779 (2006)

P. Maunz *et al.*, Nature Physics 3, 538 (2007)

E. B. Flagg *et al.*, PRL 104, 137401 (2010)

# The anyon collider, Rosenow et al., Phys. Rev. Lett **116**, 156802 (2016)

Chiral Luttinger liquid description: bosonic fields describe charge fluctuations at the input of the collider



$$V_1 = V_2 = V_S$$

$$T_1 = T_2 = T_S \ll 1$$

$$I_+ = I_1 + I_2 \neq 0$$

$$I_- = I_1 - I_2 = 0$$

$$[\phi_i(0, t), \phi_j(0, t')] = i\pi\delta_{ij} \delta(\varphi) sgn(t - t')$$

$$H_T = \zeta e^{i\phi_1(0,t)-i\phi_2(0,t)} + h.c.$$

Poissonian emission of quasiparticles at inputs 1 and 2:

$$\phi_i(0, t) = \phi_i^{(0)}(0, t) + 2\pi\lambda N_i(t)$$

$N_i(t)$ : random (poissonian) variable, number of quasiparticles emitted in time  $t$

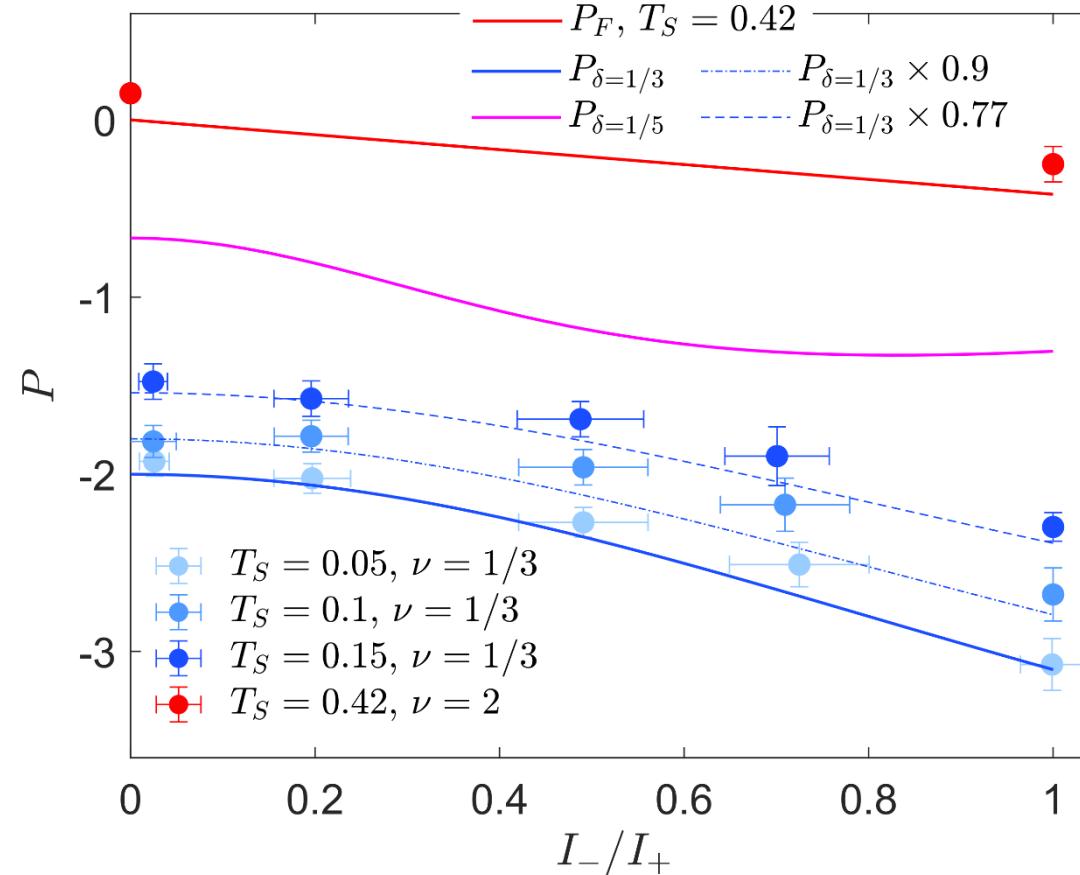
Laughlin case:  $\lambda = 1/m$

but  $\lambda$  can be renormalized by interactions

$$S_{I_3 I_4} = P 2qT I_+, \quad P = 1 - \frac{\tan(\pi\lambda)}{\tan(\pi\delta)} \frac{1}{1 - 2\delta} = -2 \quad (\lambda = \delta = v = 1/3)$$

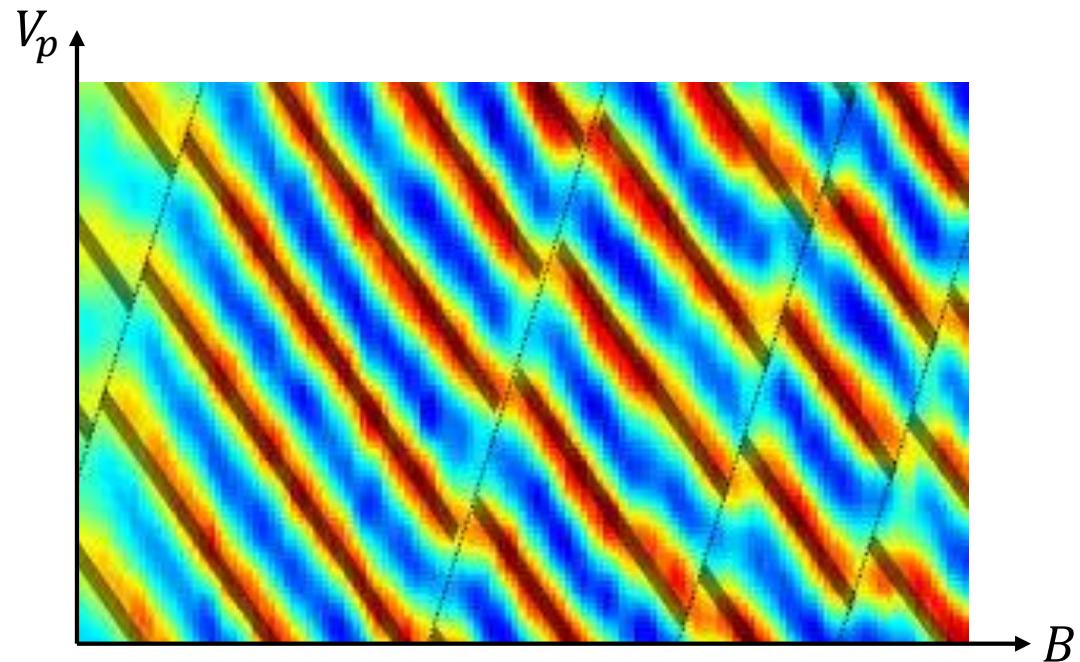
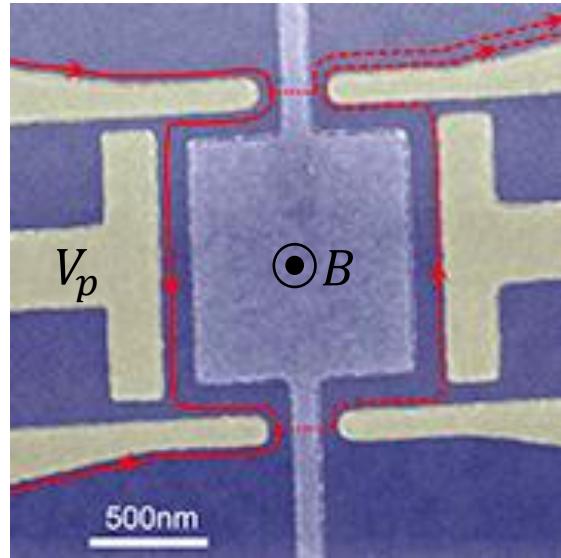
# Anyon collisions, $I_- \neq 0$

$$T_1 = T_2 = T_S \quad V_1 \neq V_2 \quad \left\{ \begin{array}{l} I_+ = I_1 + I_2 \neq 0 \\ I_- = I_1 - I_2 \neq 0 \end{array} \right. \quad \rightarrow \quad P(I_-/I_+)$$



Very good agreement with predictions for anyon collisions with  $\varphi = \frac{\pi}{3}$

## Conclusion 2: Fabry-Perot experiments



C. de C. Chamon, et al., PRB **55**, 2331 (1997)

K.T. Law, D.E. Feldman, Y. Gefen, PRB **74**, 045319 (2006)

J. Nakamura, S. Liang, G.C. Gardner, M.J. Manfra, Nature Physics **16** 931 (2020).

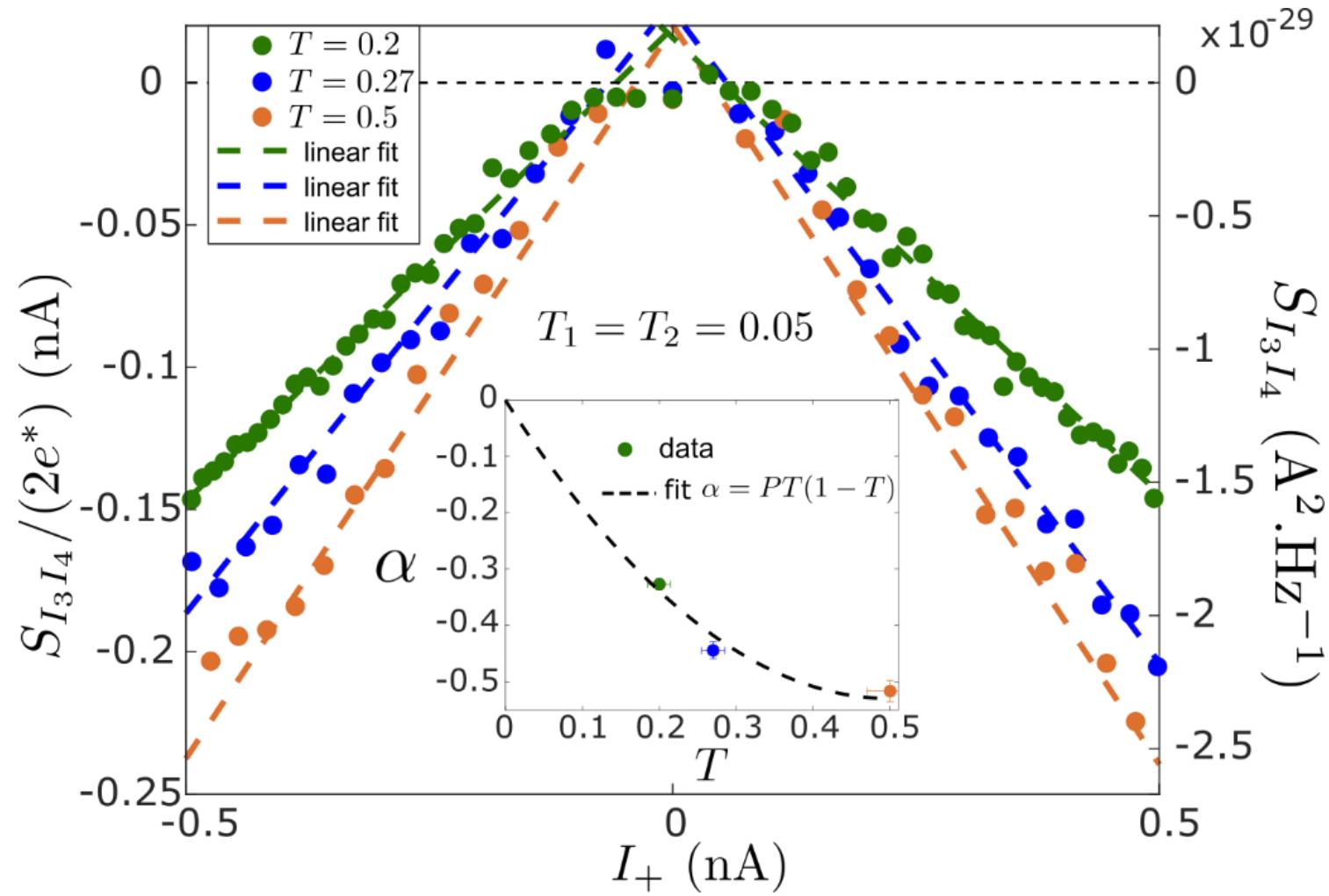
$$\theta = 2\pi \frac{q}{h} BA + N_{qp} 2\varphi$$

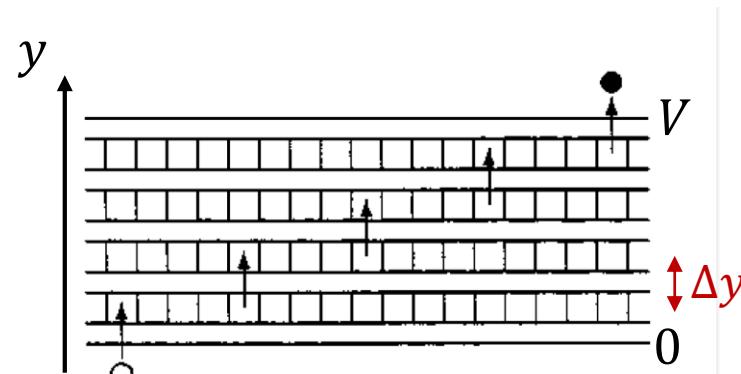
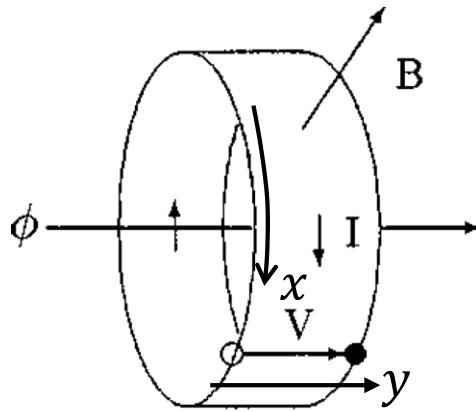
AB phase
Braiding phase

$$\nu = 1/3$$

$$2\varphi = 2\pi/3$$

Important: coulomb interactions can be neglected





$$\vec{A} = By\vec{e}_x$$

$$H = \sum_{j=1}^N \frac{(\vec{p}_j - e\vec{A}(\vec{r}_j))^2}{2m} + eEy_j$$

$$\psi_{k,n}(x, y) = e^{ik_p x} \phi_n(y + y_0 - \frac{\hbar k_p}{eB}) \quad y_0 = \frac{Em}{eB^2} \quad k_p = p \frac{2\pi}{L_x} \rightarrow \Delta y = \frac{h}{eBL_x}$$

$$\vec{A} \rightarrow \vec{A} + A_0 \vec{e}_x$$

$$\Delta\phi = A_0 L_x$$

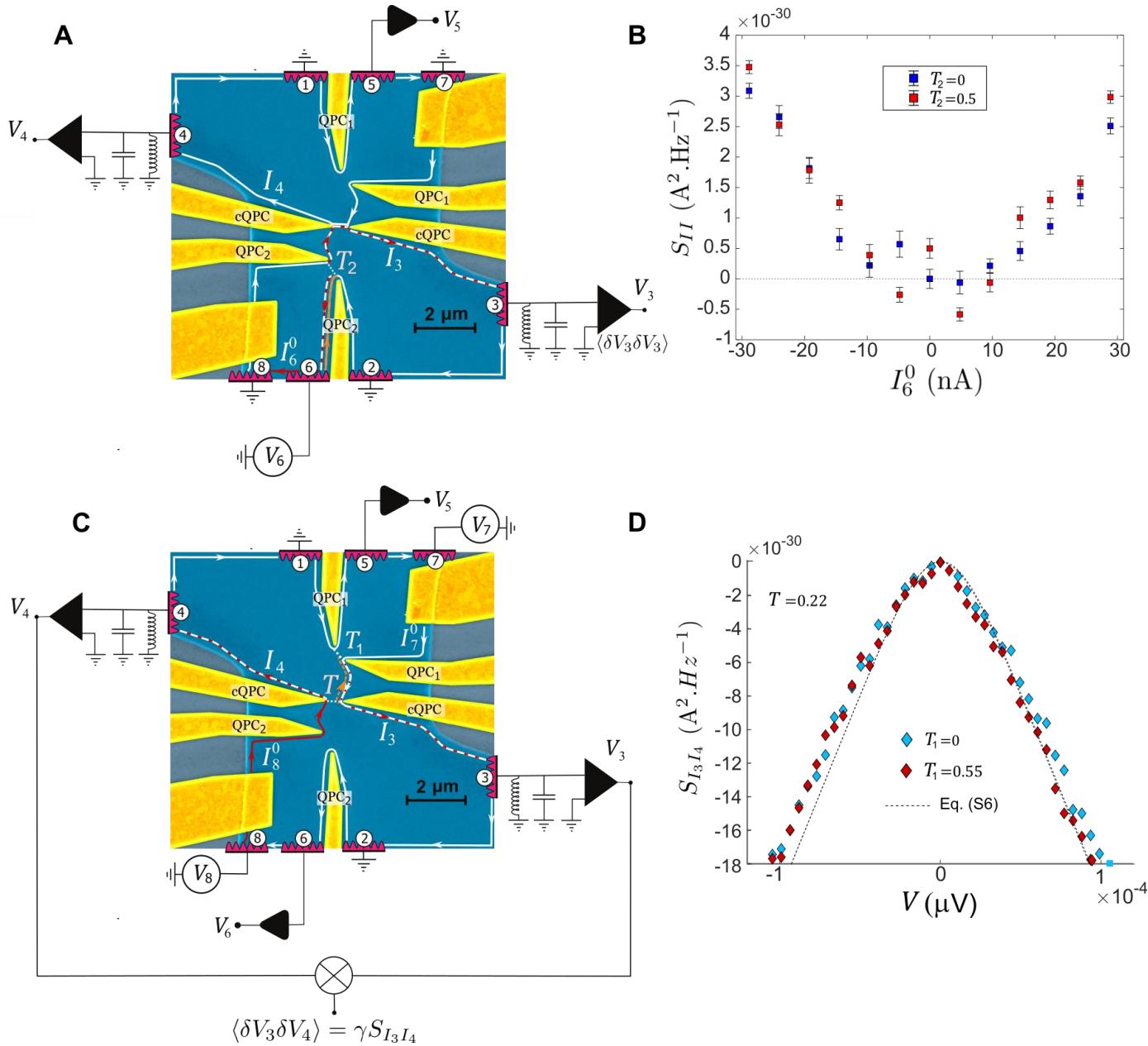
$$\psi_{k,n}(x, y) \rightarrow e^{ik_p x} \phi_n(y + y_0 - \frac{\hbar k_p}{eB} - \alpha) \quad \alpha = \frac{A_0}{B}$$

For  $\alpha = \Delta y, \Delta\phi = h/e$

wavefunctions shifted to another:  
one electron is transferred

$$I = \frac{\Delta H}{\Delta\phi} = \frac{qV\nu}{h/e} \rightarrow \begin{cases} q = e, G = \nu \frac{e^2}{h} \\ q = e/3, G = \frac{e^2}{3h} \end{cases}$$

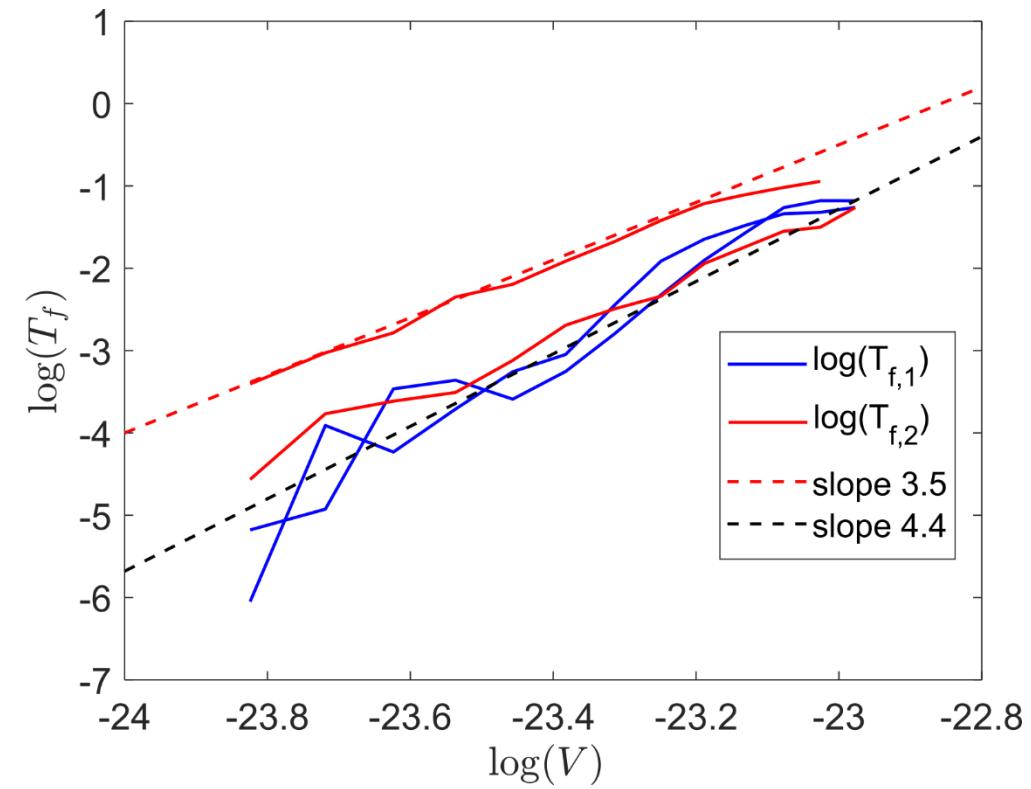
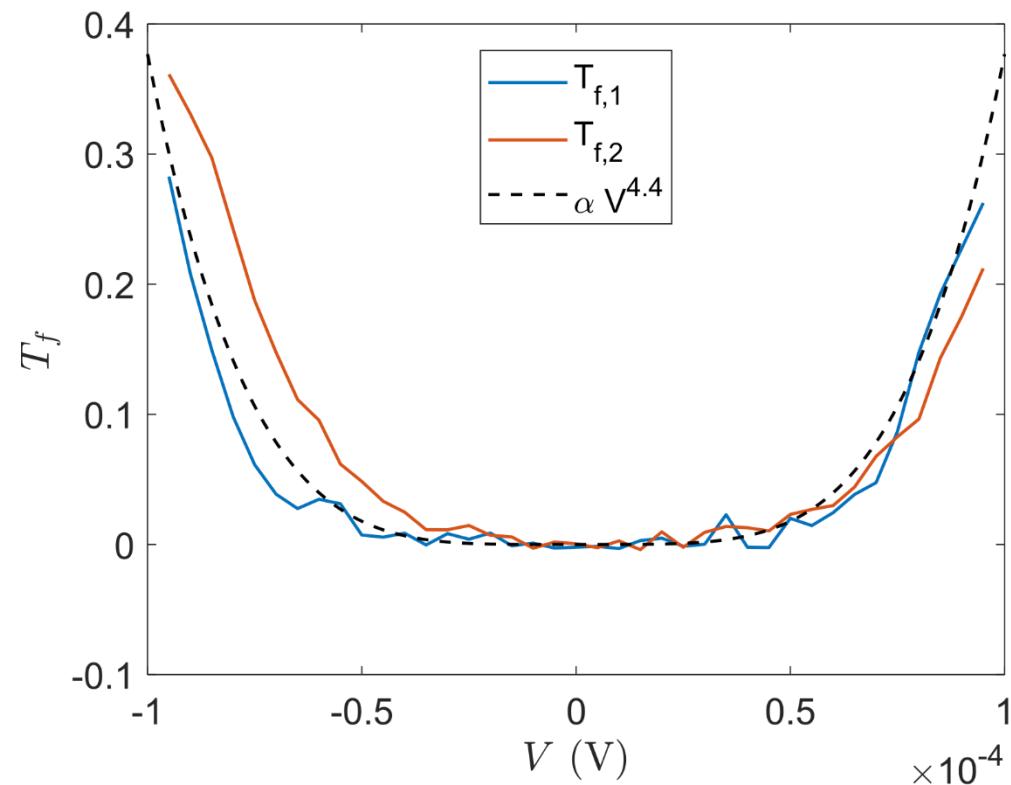
# Looking for neutral modes?



# Non-linearities, weak tunneling limit

Weak tunneling limit:  $T_f = 1 - T \ll 1$

Pinched QPC



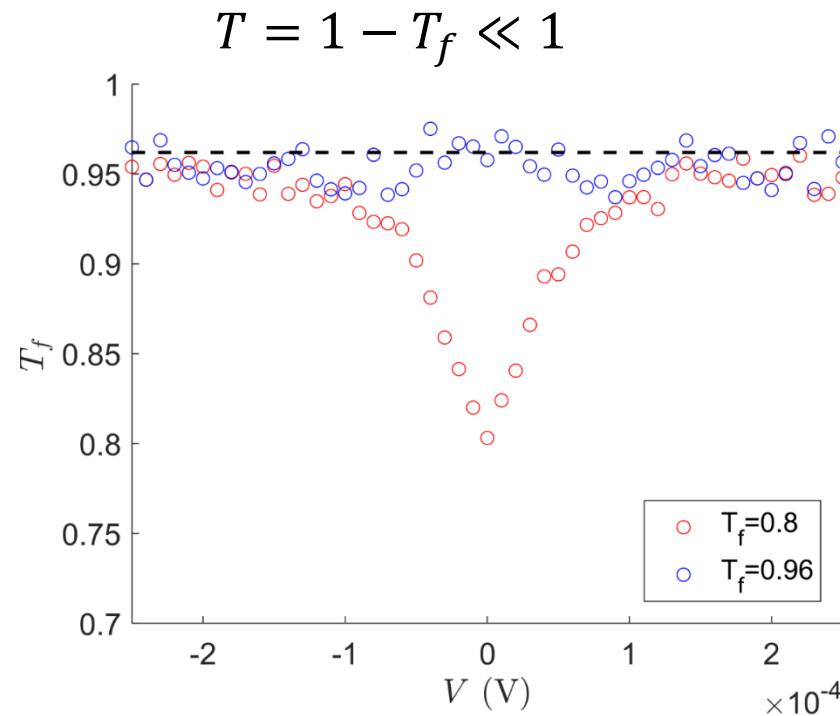
$$T_f = \frac{\partial I_f}{\partial V} \frac{3h}{e^2} \propto V^{\frac{2}{\delta}-2}$$

$$3.5 \leq \frac{2}{\delta} - 2 \leq 4.5$$

$$0.36 \leq \delta \leq 0.31$$

# Non-linearities, weak backscattering limit

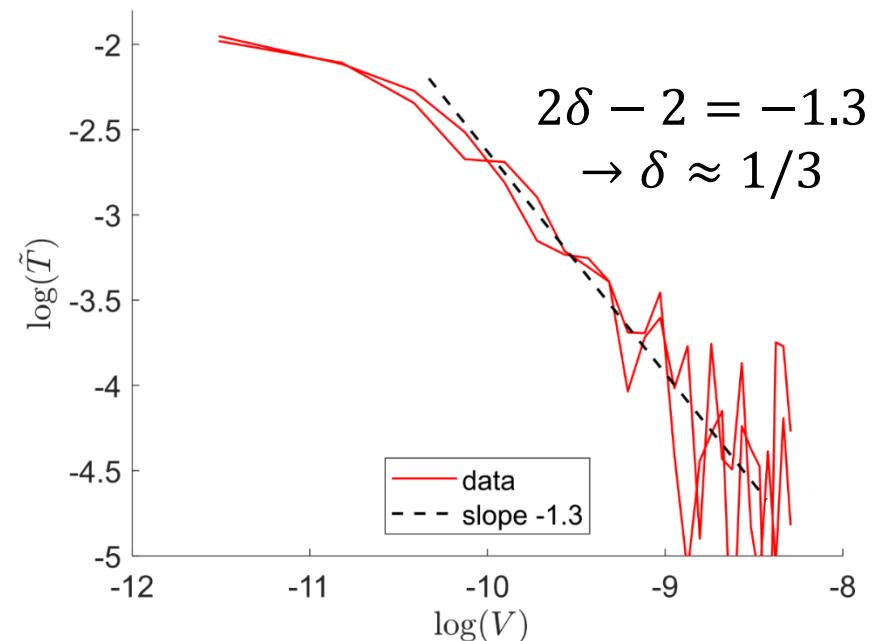
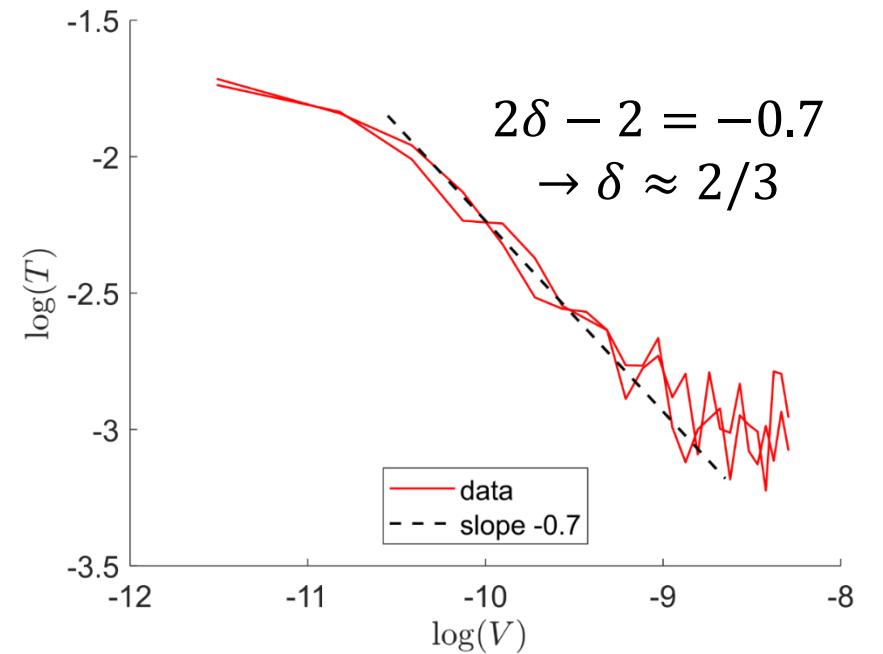
Weak backscattering limit: Open QPC



$$T = \frac{\partial I_b}{\partial V} \frac{3h}{e^2} \propto V^{2\delta-2}$$

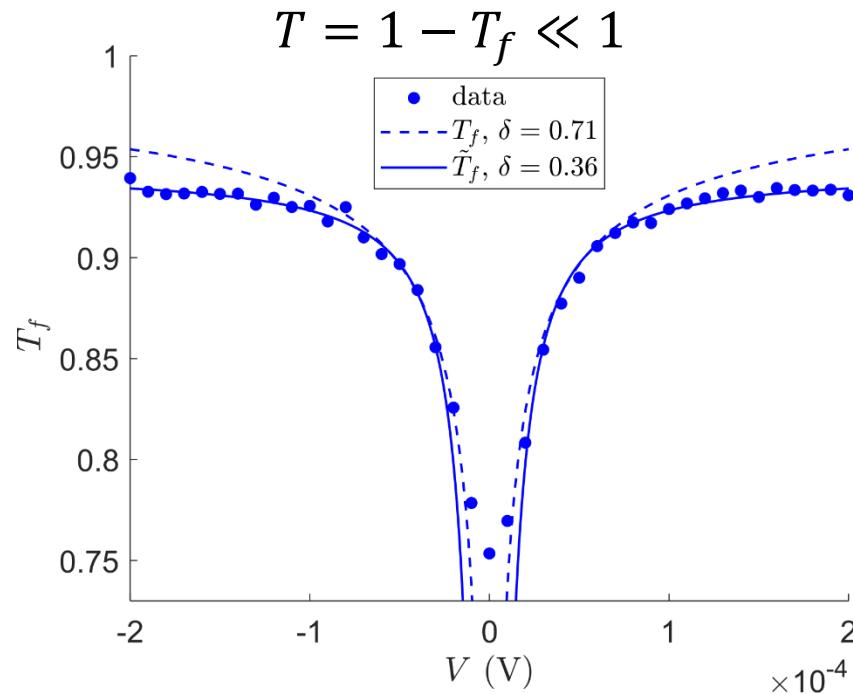
$$\tilde{T} = 0.96 - T_f$$

$$\tilde{T} \propto V^{2\delta-2}$$



# Non-linearities, weak backscattering limit

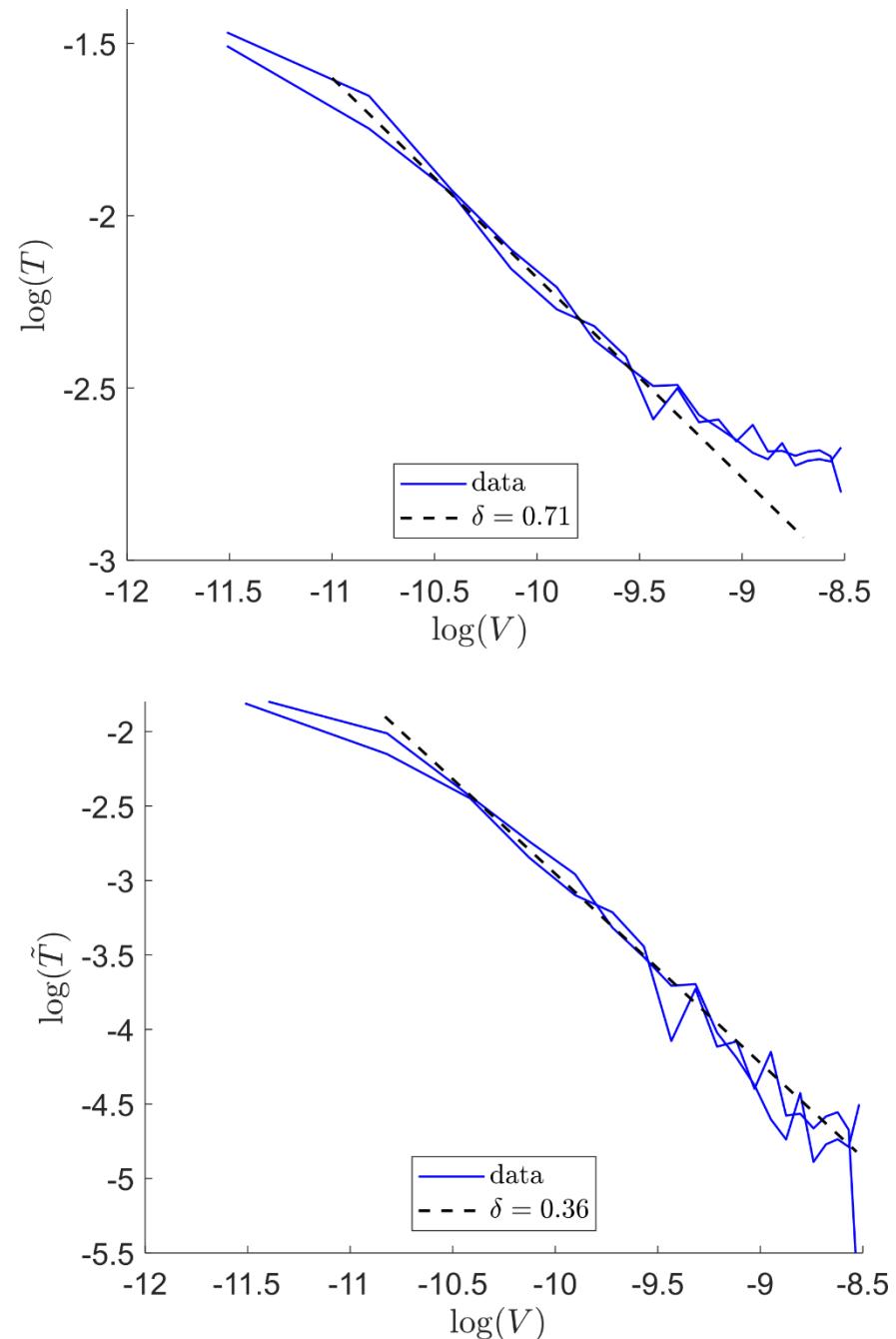
Weak backscattering limit:



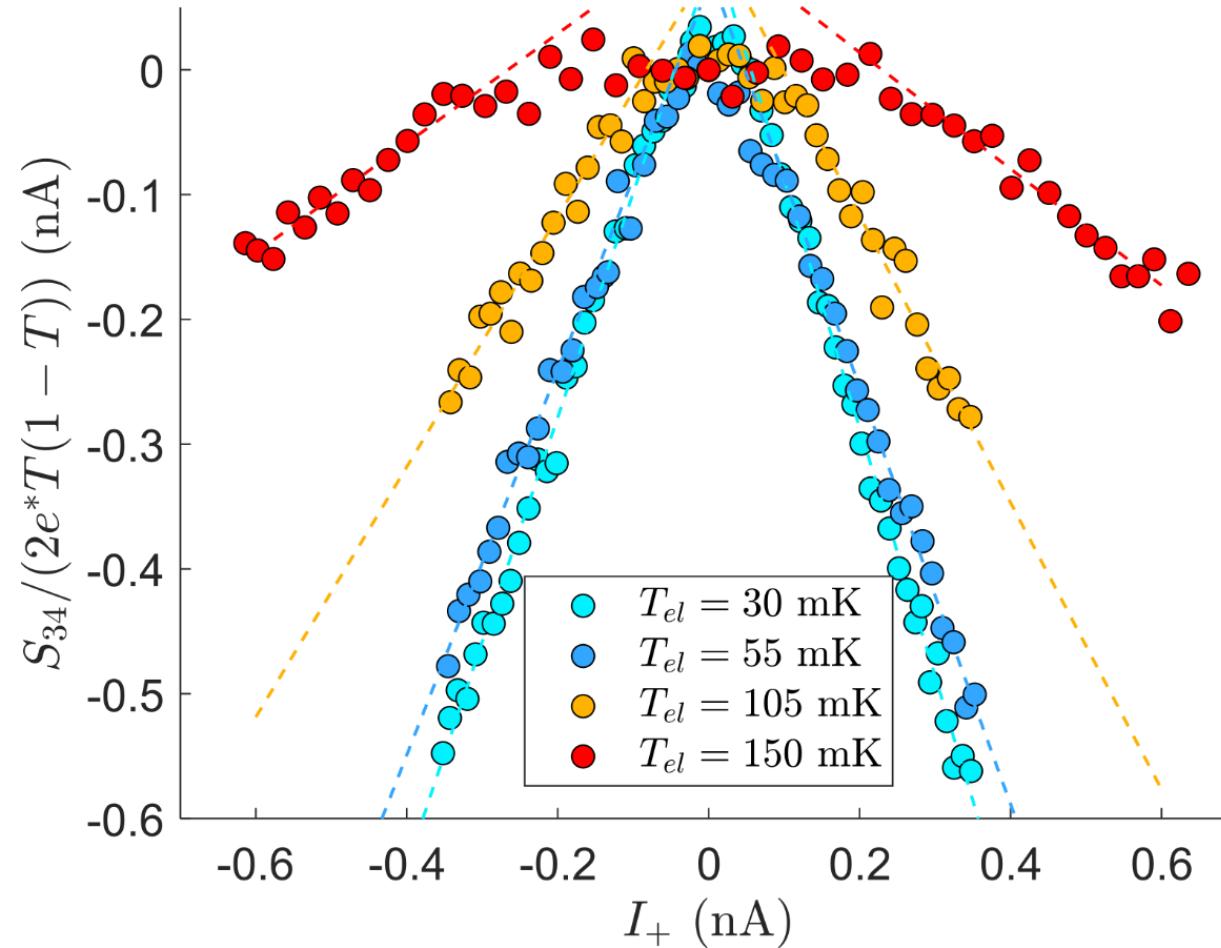
$$T_f = \frac{\partial I_b}{\partial V} \frac{3h}{e^2} \propto V^{2\delta-2}$$

$$\tilde{T}_f = 0.942 - T_f$$

$$\tilde{T}_f \propto V^{2\delta-2}$$



## Temperature dependence



# The collider: sample

